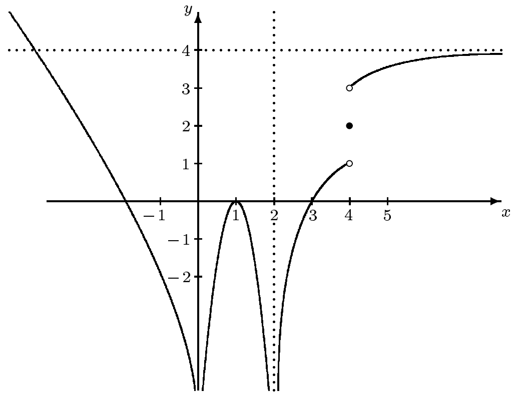
CEGEP JOHN ABBOTT COLLEGE

# Calculus I (Science)

(Mathematics 201–NYA/??)



1. Consider the function $y = f(x)$ whose graph is given below:




Evaluate the followng (if an item does not exist, please be specific: assign one of $\infty$ or $-\infty$, or state that it does not exist).

(a) The $x$ value(s) for which $f$ is discontinuous. (b) $f(4)$

(c) $\lim\limits_{x\to-\infty} f(x)$ (d) $\lim\limits_{x\to-1^-} f(x)$ (e) $\lim\limits_{x\to 0} f(x)$

(f) $\lim\limits_{x\to 2} f(x)$ (g) $\lim\limits_{x\to 4} f(x)$ (h) $\lim\limits_{x\to\infty} f(x)$

2. Using algebraic techniques and *showing your work*, find the limits:

(a) $\lim\limits_{x\to 4} \dfrac{x^2-x-12}{2x^2-9x+4}$ (b) $\lim\limits_{x\to\infty} \dfrac{x^2-x-12}{2x^2-9x+4}$

(c) $\lim\limits_{x\to 5^+} \dfrac{(x-4)(x-7)}{(x-5)(x-6)}$ (d) $\lim\limits_{x\to\frac{\pi}{6}} \dfrac{\sin 2x}{\sin x}$

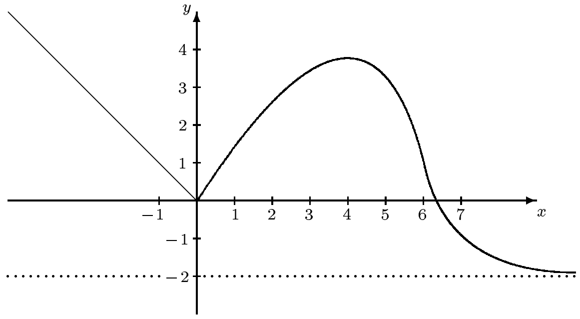
3. (a) State a limit definition of the derivative $f'(x)$ of a function $f(x)$.
   (b) Use the above definition to find $f'(x)$ given $f(x) = \dfrac{1}{x}$.

4. (a) State the requirements for a function to be continuous at $x = a$.
   (b) Using your definition above, determine if $f(x)$ is continuous at $x = 0$:

$$f(x) = \begin{cases} (x-1)^2 & \text{if } x < 0 \\ 1 & \text{if } x = 0 \\ \cos x & \text{if } x > 0. \end{cases}$$

5. Sketch, if possible, a function that is continuous, but not differentiable at $x = a$. [If this is not possible, state why not.]

6. (a) Sketch the first derivative of the function below.




   (b) Sketch the graph of a function whose first derivative is positive everywhere and whose second derivative is negative everywhere.

7. Find the derivative of each of the following functions. Simplification is not necessary.

(a) $f(x) = \dfrac{x^6}{6} + \dfrac{5}{x^2} - \sqrt[3]{x^2} + 7x - 4\pi^2$

(b) $y = x^7 \cos x$ (c) $f(t) = \dfrac{\tan t}{\ln t + 2}$

(d) $f(x) = 3e^{2x} - \sec\left(x^{1/2}\right)$ (e) $y = \sin^3\left(7 + (2x+1)^{-1}\right)$

8. Find $\dfrac{dy}{dx}$, but do not simplify:

(a) $y = \dfrac{\sqrt{x^3-6}(4x+3)^3}{x^2(3x-7)^4}$ (Use logarithmic differentiation.)

(b) $e^y + x^2 - y^2 = xy$ (Use implicit differentiation.)

9. Find an equation for the tangent line to the graph of $y = \sqrt{3x^2-2}$ at the point where $x = 3$.

10. (a) If $f(x) = e^{-\frac{x^2}{2}}$, find $f''(x)$ and simplify your answer.
    (b) Use $f''(x)$ to establish the inflection points of $f(x)$.

11. For $f(x) = x^5 - 4x^4 + 4x^3 = x^3(x-2)^2$, find $f'(x)$ and all $x$ at which $f$ has a relative maximum or relative minimum.

12. Given: $f(x) = \dfrac{72(x-2)}{x^3}$, one finds

$$f'(x) = \frac{-144(x-3)}{x^4} \qquad \text{and} \qquad f''(x) = \frac{432(x-4)}{x^5}.$$

(a) Specify on which interval(s) $f$ is concave up.
(b) Find the coordinates of relative extrema and point(s) of inflection.
(c) Sketch a graph of $f$ showing the points in (b), as well as intercepts and asymptotes.

13. Find the absolute minimum of $g(x) = \dfrac{x^2-1}{x^2+1}$ on $[-1, 1]$.

14. Find the area enclosed by $y = x^3 + 1$; $x = -1$; $x = 3$ and $y = 0$.

15. A manufacturer wants to design an open box with a square base and a surface area of 108 ft$^2$. What dimensions will produce a box with maximum volume?

16. Find the following indefinite integrals:

(a) $\displaystyle\int \left(\frac{x^2}{2} - \frac{2}{x^2} + 5e^x - \pi^2\right) dx$

(b) $\displaystyle\int \sec x(\tan x - \sec x)\,dx$ (c) $\displaystyle\int \frac{3x^3-1}{x}\,dx$

17. Evaluate, expressing your answer without decimals:

(a) $\displaystyle\int_1^4 \left(\sqrt{x}+2\right)^2 dx$ (b) $\displaystyle\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \sec^2 x\,dx$

18. Evaluate to 3 decimal places: $\displaystyle\int_1^2 \left(e^x + \frac{1}{x}\right) dx$.

19. A spherical Tootsie Roll Pop you are sucking on is giving up volume at a steady rate of 80 mm$^3$/min. How fast will the radius be decreasing when the Tootsie Roll Pop is 20mm in diameter? ($V_{sphere} = \frac{4}{3}\pi r^3$)