(Marks)

(12) 1. Consider the curve given by the equations

$$x = t - 2t^2 \quad \text{and} \quad y = t^2 + 3t$$

(a) Find the t-values and the coordinates of the x-intercepts.

(b) Find $\frac{dy}{dx}$. Use $\frac{dy}{dx}$ to sketch the graph of the curve between the x-intercepts.

(c) Find the area bound by the curve and the x-axis.

(d) Set up, *but do not evaluate*, an integral that gives the arc length of the curve between the x-intercepts.

(7) 2. (a) Sketch the graphs of $r = 1 + \cos\theta$ and $r = 3\cos\theta$, on the same axes.

(b) Find the area of the region that lies *outside* the cardioid $r = 1 + \cos\theta$ and *inside* the circle $r = 3\cos\theta$.

(9) 3. (a) Find the Maclaurin series for $\displaystyle\int_0^x \frac{t^2\,dt}{1+t^4}$.

(b) What is the interval of convergence for this power series?

(c) Use the answer to 3(a) to approximate $\displaystyle\int_0^{1/2} \frac{t^2\,dt}{1+t^4}$ to within an error of $\pm 10^{-4}$. (Justify your approximation.)

(6) 4. (a) Find the Maclaurin series for $\dfrac{x - \sin x}{x^3}$.

(b) Use the series (from 4(a)) to calculate the limit $\displaystyle\lim_{x\to 0} \frac{x - \sin x}{x^3}$. Verify your answer using L'Hôpital's rule.

(6) 5. For the surface $x\tan z = y$ find the equation of the tangent plane at the point $P_0\,(1, 1, \frac{\pi}{4})$. Find the parametric equations for the normal line through the same point P_0.

(6) 6. Let $f(x, y, z) = x\,\mathrm{e}^{yz}$.

(a) Find the directional derivative of f in the direction $\boldsymbol{i} + \boldsymbol{k}$ at the point $P_0\,(2, 0, -4)$.

(b) Find the direction in which f is *decreasing* at a maximal rate at the same point P_0, and what is the rate of the decrease in that direction?

(10) 7. A curve is defined by $\boldsymbol{r}(t) = \langle t^2, \sin t - t\cos t, \cos t + t\sin t\rangle$, $t > 0$. Find the unit tangent and unit normal vectors $\boldsymbol{T}, \boldsymbol{N}$, and the curvature κ. Find the tangential and normal components of acceleration $a_{\boldsymbol{T}}, a_{\boldsymbol{N}}$.

(7) 8. Let $u = u(x, y)$, $x = r\cos\theta$, $y = r\sin\theta$. Show that

$$\left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2 = \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2$$

(7) 9. Find and classify the critical points of $f(x, y) = x^3 - 3xy + y^3$.

(8) 10. Use *two* Lagrange Multipliers to find the maximum value of $f(x, y, z) = xy + yz$ on the line of intersection of the two planes $x + 2y = 6$ and $x - 3z = 0$ (*i.e.* subject to the constraints imposed by those two equations).

(Marks)

(15) 11. Evaluate the following.

(a) $\displaystyle\int_0^2 \int_{y^2}^4 \frac{\sin x}{\sqrt{x}}\, dx\, dy$

(b) $\displaystyle\int_0^1 \int_0^{\sqrt{1-y^2}} \mathrm{e}^{x^2+y^2}\, dx\, dy$

(c) $\displaystyle\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \sqrt{x^2+y^2+z^2}\, dz\, dy\, dx$

(7) 12. Find the volume of the solid bounded by $y = x$, $y = 2$, $x = 0$, the xy-plane, and $z = 4 - y^2$.