CALCULUS III FINAL EXAMINATION FALL 1997

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INSTRUCTIONS :
1 ATTEMPT ANY 12 (of 13) PROBLEMS .
Indicate clearly which problem should not be marked .
2 Each complete question caries equal weight .
3 You may assume the following :

For all x : $\sin x = \sum_{k=0}^{\infty} \dfrac{(-1)^k x^{2k+1}}{(2k+1)!}$ $\quad \cos x = \sum_{k=0}^{\infty} \dfrac{(-1)^k x^{2k}}{(2k)!}$ $\quad e^x = \sum_{k=0}^{\infty} \dfrac{x^k}{k!}$

For $|x| < 1$: $\dfrac{1}{1-x} = \sum_{k=0}^{\infty} x^k$

ATTEMPT ANY 12 (COMPLETE) PROBLEMS

1 Obtain the Maclaurin series for $f(x) = \dfrac{1}{2}\sin 2x$ by :
a substitution in the Maclaurin expansion for $\sin x$.
b multiplying the series expansions of $\sin x$ and $\cos x$
(first four non-zero terms only)
(since $\sin 2x = 2\sin x \cos x$)
1c *OPTIONAL (BONUS 5%)*
Use any other method other than those of **a** and **b** above to obtain the Maclaurin series for $f(x) = \dfrac{1}{2}\sin 2x$.

2 Approximate $\int_0^1 e^{-x^2}\, dx$ to 6 decimal places .

3a Establish a Maclaurin series expansion for $f(x) = \ln(1+x^2)$
b Find $f^{(7)}(0)$.

4 Given $r = 2 - 2\sin\theta$; $r = 2\sin\theta$
a Sketch both graphs on the same set of axes .
b Find all points of intersection .
c Write an integral for the area of the region common to both .
DO NOT ATTEMPT TO EVALUATE THE INTEGRAL.

5 Consider the space curve $\overrightarrow{R}(t) = \overrightarrow{(2\cos t, 2\sin t, t)}$
a Draw a rough sketch of the curve. $(0 \le t \le 2\pi)$
b Calculate the arc length. $(0 \le t \le 2\pi)$
c Find $\widehat{T}$, $\widehat{N}$, a_T , a_N and κ .

6 The period T of a simple pendulum of length l and gravitational constant g is computed using the formula $T = 2\pi\sqrt{\frac{l}{g}}$
l was measured as $5.00 \pm .15$ *metres*
g has been determined $9.75 \pm .10$ *metres*
Using DIFFERENTIALS , estimate the maximum possible error in T .

7 Determine the minimum distance from $(1, -1, 1)$ to the sphere $x^2 + y^2 + z^2 = \frac{1}{3}$.

8 Find and classify all the critical points of $f(x, y) = x^3 + y^3 - 3x - 12y + 20$

9 $w = f(u) + g(v)$; $u = x - at$; $v = x + at$
Show that: $\dfrac{\partial^2 w}{\partial t^2} = a^2 \dfrac{\partial^2 w}{\partial x^2}$

10 Write integrals for the volumes of the following regions:
DO NOT ATTEMPT TO EVALUATE THE INTEGRALS
a the region enclosed by $z = x^2 + y^2$ and $z = 18 - x^2 - y^2$
b the region bounded by the planes $z = 0$, $z = x + y$, $y = 2x$ $y = 2$ and $x = 0$.

11 Evaluate $\int_0^1 \int_x^1 x \sin(y^3)\, dy\, dx$ (Hint: Reverse the order of integration.)

12 Find the centroid of the homogenous solid region [(shaped like an ice cream cone (1 scoop)] bounded by the sphere $x^2 + y^2 + z^2 = R^2$ and the cone $z = \sqrt{x^2 + y^2}$.

13 Establish a formula for the area of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$
Hint : Start with a double integral and the transformation $u = \dfrac{x}{a}$, $v = \dfrac{y}{b}$.