Final Examination Calculus III

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1. Obtain the Maclaurin series for $g(x) = \sin 2x$
   (a) by substitution in the Maclaurin expansion for $\sin x$. (3%)
   (b) using the identity $\sin 2x = 2 \sin x \cos x$ and multiplying the series expansions for $\sin x$ and $\cos x$. (first 4 non-zero terms only) (3 %)
   (c) using a method other than those of a) and b) above . (3%)
2. By inspection ,evaluate :
   (a) $1 - \frac{\pi^2}{2!} + \frac{\pi^4}{4!} - \frac{\pi^6}{6!} + \frac{\pi^8}{8!} - + \ldots$ (2 %)
   (b) $\sum_{k=1}^{\infty} \frac{1}{k!}$ (2%)
3. Given $f(x) = \sum_{k=1}^{\infty} \frac{(-1)^k (x+3)^k}{k(k!)}$ Find $f^{(10)}(-3)$ (3 %)
4. Let $f(x) = \ln(1 + x^2)$
   (a) Find a Maclaurin series for the derivative of f $i.e.$ $f'(x)$ (3 %)
   (b) Use the above result to obtain a series for $f(x)$ (Provide a formula for the general term .) (3 %)
   (c) Use the series just obtained to approximate $\ln(1.01)$ with error less than 10^{-6} . (3 %)

5. Consider the space curve $\overrightarrow{R}(t) = \overrightarrow{(2\cos t, 2\sin t, t)}$

(a) Draw a rough sketch of the curve.$(0 \leq t \leq 2\pi)$ (2 %)

(b) Write the parametric equations of the tangent line to the curve at $P(2,0,0)$. (2 %)

(c) Calculate the length of the curve from $P(2,0,0)$ to $Q(-2,0,\pi)$. (2 %)

(d) Find a_T , a_N , $\overrightarrow{T}$ and $\overrightarrow{N}$. (4 %)

(e) Determine the curvature at $P(2,0,0)$. (2 %)

6. Demonstrate that the curve C whose vector equation is $\overrightarrow{R}(t) = \ln t\ \overrightarrow{i} + (t^2-1)\overrightarrow{j} + t\ \overrightarrow{k}$ is tangent to the surface $S:\ xz^2 - yz + \cos(xy) = 2$ at the point $P(0,-1,1)$. (3 %)

7. Roughly sketch the following surfaces and include all intercepts. State the name of each surface.

(a) $x^2 + y^2 = z^2 + 9$ (3 %)

(b) $z^2 = 9 - 4x^2 - y^2$ (3 %)

8. Given : $r = 2 - 2\sin\theta$; $r = 2\sin\theta$

(a) Sketch both graphs on the same set of axes. (2 %)

(b) Find all points of intersection. (2 %)

(c) Write integrals

for i) the area of the region common to both. (3 %)

ii) the length of the part of $r = 2 - 2\sin\theta$ outside $r = 2\sin\theta$. (3 %)

DO NOT ATTEMPT TO EVALUATE

9. Evaluate:

(a) $\displaystyle\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4}\, dx dy$ (4 %)

(b) $\displaystyle\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^2 (x^2+y^2)\, dz\, dy dx$ (4 %)

10. Write integrals for each of the following:

DO NOT ATTEMPT TO EVALUATE THE INTEGRALS.

(a) the volume of the region enclosed by
$z = x^2 + y^2$ and $z = 18 - x^2 - y^2$. (3 %)

(b) the volume bounded by the planes $z = 0$, $z = x + y$,
$y = 2x$, $y = 2$ and $x = 0$. (3 %)

(c) the volume of the region below $x^2 + y^2 + z^2 = 9$
and above $z = \sqrt{x^2 + y^2}$. (3 %)

11. Write in spherical coordinates $(x^2 + y^2 + z^2)^2 = 2z(x^2 + y^2)$ (3 %)

12. Find the maximum and minimum values of $f(x, y) = xy$
on the circle $x^2 + y^2 = 1$ **USING LAGRANGE MULTIPLIERS**. (6 %)

13. Find and classify the critical points of $f(x, y) = x^3 + 3xy^2 + 3y^2 - 15x + 2$ (6 %)

14. Given $y^2 z e^{x+y} - \sin(3z) = 0$, find $\dfrac{\partial z}{\partial y}$. (3 %)

15. Given $z = x^2 \sin y$, $x = s^2 + t^2$, $y = s^2 t$, find $\dfrac{\partial z}{\partial s}$ (4 %)
using the Chain Rule.

16. Find the direction and rate of maximum increase of
$f(x, y, z) = x^2 e^y - x \ln(y^2 + z^2)$ at $(1, 0, 1)$. (5 %)