(Marks)

(2×1)   1. Use the laws of exponents to simplify the expressions below. Give your answers using only positive exponents.

     (a) $(2a^{-2}b^2)^{-1}(2a^3b^{-3})^2$ (b) $\sqrt[3]{\dfrac{2^4x^7y^2}{2^{-2}xy^5}}$

(2)   2. Rationalize the denominator, and simplify: $\dfrac{2}{3+\sqrt{11}}$

(3)   3. Perform the long division: $\dfrac{2x^4-9x^3+13x+7}{2x+3}$

(2×3)   4. Perform the indicated operations and simplify the result:

     (a) $\dfrac{x^2-16}{x^2+5x+4}\div\dfrac{x^2-8x+16}{x^2-1}$ (b) $\dfrac{3x}{x-1}-\dfrac{x+1}{x+2}$

(3×3)   5. Factor completely.

     (a) $4x^4+15x^3-4x^2$ (b) x^9+8x^6 (c) $12a^2+4a-3ab-b$

(3×2)   6.   (a) Find the equation of the straight line through $\left(\frac{3}{4},\frac{1}{2}\right)$ and $\left(\frac{3}{4},-\frac{1}{2}\right)$.

     (b) Find the equation of the straight line through the origin $(0,0)$ with slope $\frac{2}{3}$.

     (c) Find the equation of the straight line through $(2,-4)$ parallel to the line $3y=x-4$

(2×1)   7. Given points $A(-2,3)$ and $B(2,1)$:

     (a) Find the midpoint of AB.   (b) Find the length of the line segment AB.

(3)   8. Given $f(x)=x^2-x$, find and simplify $\dfrac{f(x+h)-f(x)}{h}$. Evaluate the simplified form when $h=0$.

(3×1)   9. If $f(x)=\dfrac{1}{x}$ and $g(x)=x+2$, find:

     (a) $f(g(x))$ (b) $g(f(x))$ (c) $f(x)+g(x)$

(4×3)   10. Solve:

     (a) $3x^2-5x=1$ (b) $\sqrt{2x+8}=x$ (c) $\dfrac{x}{x+1}-\dfrac{2}{x}=\dfrac{1}{x^2+x}$ (d) $x+3\geq 2x-7$

(2)   11. Find the inverse of $f(x)=\dfrac{x+1}{x-1}$.

(2×1)   12. Give the exact values of the following.

     (a) $\log_2 16$ (b) $3+\mathrm{e}^{2\ln 3}$

(2)   13. If you invest \$5000 at 6% compounded semi-annually, how much will your investment be worth at the end of 10 years?

(4×3)   14. Sketch the graph of each of the following, giving the required information.

     (a) $y=-x^2+8x-12$.   Give: domain, range, vertex, x and y intercepts.

     (b) $y=\dfrac{1-x}{3x+1}$.   Give: x and y intercepts, vertical and horizontal asymptotes.

     (c) $y=\begin{cases} 1-2x & \text{if } x<2 \\ x^2 & \text{if } x\geq 2 \end{cases}$.   Give: the domain, range, x and y intercepts.

     (d) $y=2\sin(\pi x)$.   Give: the amplitude and period. Sketch two cycles.

(Marks)

(4×3) 15. Solve for x:

  (a) $\log(x+8) - \log(x-1) = 1$ (b) $\log_x\left(\frac{1}{16}\right) = 4$

  (c) $2^{3x+1} = 7$ (d) $\left(\frac{1}{2}\right)^x = 8^{2-x}$

(3×1) 16. Approximate to 4 decimal places:

  (a) $\cos(2.3)$ (b) $\csc(52.8°)$ (c) $\tan^{-1}(6.65)$

(2×1) 17. Convert:

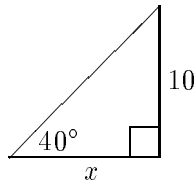 (a) $240°$ to radians (b) $\frac{5\pi}{12}$ to degrees

(1) 18. If $\sin\theta = -\frac{\sqrt{3}}{2}$ and θ lies in the third quadrant, find $\tan\theta$.

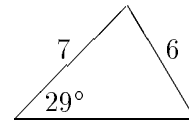(2) 19. Find all θ between 0 and 2π which satisfy: $\sin\theta = -\frac{1}{2}$.

(2×3) 20. Prove the following identities:

  (a) $\sec x - \cos x = \sin x \tan x$ (b) $\cos^4\theta + \cos^2\theta\sin^2\theta = \cos^2\theta$

(2) 21. Find x:

(3) 22. Find the missing angles and side length for the following
triangle. (If more than one answer is possible, give all answers.)

(3) 23. Angie is 5 meters away from Bobby and 10 meters away from Chris. If she looks straight at Bobby, she must turn $108°$ to look straight at Chris. How far apart are Bobby and Chris?

Answers

1.(a) $\frac{2a^8}{b^8}$ (b) $\frac{4x^2}{y}$ **2.** $\sqrt{11} - 3$ **3.** $x^3 - 6x^2 + 9x - 7 + \frac{28}{2x+3}$ **4.**(a) $\frac{x-1}{x-4}$ (b) $\frac{2x^2+6x+1}{(x-1)(x+2)}$
5.(a) $x^2(4x-1)(x+4)$ (b) $x^6(x+2)(x^2-2x+4)$ (c) $(4a-b)(3a+1)$ **6.**(a) $x = \frac{3}{4}$ (b) $y = \frac{2}{3}x$
(c) $y = \frac{1}{3}x - \frac{14}{3}$ **7.**(a) $(0,2)$ (b) $2\sqrt{5}$ **8.** $2x + h - 1 = 2x - 1$ if $h = 0$. **9.** (a) $\frac{1}{x+2}$ (b) $\frac{1}{x} + 2$
(c) $\frac{1}{x} + x + 2$ **10.**(a) $\frac{1}{6}(5 \pm \sqrt{37})$ (b) 4 (c) 3 (d) $x \le 10$ **11.** $y = \frac{x+1}{x-1}$ **12.**(a) 4 (b) 12
13. \$9030.56 **14.**(a) Dom: all x; Rng: all $y \le 4$; vertex: $(4,4)$; x-int: $(2,0), (6,0)$; y-int: $(0,-12)$.
(b) VA: $x = -\frac{1}{3}$; HA: $y = -\frac{1}{3}$; x-int: $(1,0)$; y-int: $(0,1)$. (c) Dom: all x; Rng: all $y > -3$; x-int:
$(\frac{1}{2}, 0)$; y-int: $(0,1)$. (d) Amp: 2; Period: 2. [All graphs below.] **15.**(a) 2 (b) $\frac{1}{2}$ (c) $\frac{1}{3}\left(\frac{\log 7}{\log 2} - 1\right)$
(d) 3 **16.**(a) -0.6663 (b) 1.2554 (c) $81.4482° = 1.4215$ **17.**(a) $\frac{4\pi}{3}$ (b) $75°$ **18.** $\sqrt{3}$ **19.** $\frac{7\pi}{6}, \frac{11\pi}{6}$
20.(a) LHS $= \frac{1}{\cos x} - \cos x = \frac{1-\cos^2 x}{\cos x} = \frac{\sin^2 x}{\cos x} = \sin x \frac{\sin x}{\cos x} =$ RHS (b) LHS $= \cos^2 x(\cos^2 x + \sin^2 x) =$
RHS **21.** 11.92 **22** Two answers: (i) third side: 11.1, remaining angles: $34.4°, 116.6°$ and
(ii) third side: 1.2, remaining angles: $145.6°, 5.4°$ **23.** 12.5 m apart

The Graphs:

14(a) 14(b) 14(c) 14(d)