CEGEP JOHN ABBOTT COLLEGE

INTRODUCTION TO ALGEBRA

(MATHEMATICS 201-007/∞)

1. Find the exact value of each of the following *Show your work and do not give decimal answers*.
   (a) $\frac{3}{2} \cdot \frac{8}{15} \div 2\frac{6}{7}$ (b) $\frac{-5^0(-3)^3+48}{5|8-23|}$

2. Simplify. *Give answers with positive exponents.*
   (a) $\left(\frac{x^4y^2}{y^4z^2}\right)^{\frac{1}{2}}$ (b) $\frac{(2x^2y^3)^3(3x^{-2}y^{-1})}{x^{-3}y^{10}}$

3. Perform the indicated operations and simplify.
   (a) $3y^2-2y+5-3y(4-y)$ (b) $(3x+2)(x-1)$
   (c) $(2x+1)(x^2-5x+7)$ (d) $\frac{x^3-2x^2-5x+9}{x^{+}2}$

4. Factor completely.
   (a) $x^2-7x+10$ (b) $8x^2-4xy+6xt-3yt$
   (c) $3x^2+8x-3$ (d) $2x^2-5xy+3y^2$
   (e) x^3-9x

5. Perform the indicated operations and write each answer in lowest terms.
   (a) $\frac{y^2-4}{2y^2+7y+6} \cdot \frac{2y+3}{y^2-2y}$ (b) $\frac{2w^2+6w}{w^2+2w-3} \div \frac{2w(w+3)}{1-w}$
   (c) $\frac{4}{x^2-x-2} - \frac{1}{(x-2)(x-1)}$ (d) $\frac{2}{x-1} + \frac{1}{x} - \frac{3}{x^2}$

6. Solve the following equations.
   (a) $-2(2x-1)-8 = 3(x+2)+2$ (b) $2(3x-2)-1 = 6x-5$
   (c) $x(x-10) = -25$ (d) $(2x-1)^2 = 121$
   (e) $x^2 = x+1$ (f) $4x^2+4x+3 = 0$
   (g) $\frac{x+2}{x} = \frac{3x-2}{2}$ (h) $\frac{3}{x} - \frac{1}{x+1} = \frac{5}{x(x+1)}$

7. Using algebraic techniques and defining variables, solve the following problems.
   (a) If three times a number decreased by four is equal to two, find the number.
   (b) A car has a price of \$8640. For trading in his old car, Tom will get 30% off. Find the price of the car with the trade in.
   (c) A ladder is 17 feet long. It reaches 15 feet up on a wall. How far is the base of the ladder from the wall?

8. Given the line equation $5x+3y=15$
   (a) find the x-intercept, the y-intercept and the slope.
   (b) use the x- and y-intercept to graph the line.

9. Give an equation for each of the following the lines:
   (a) with slope $\frac{4}{3}$ and passing through the point $(2,-3)$.
   (b) passing through the points $(3,-1)$ and $(-5,-3)$.

10. Given the equations for $\begin{cases} \ell_1: & 4x + 3y = -1, \\ \ell_2: & y = \frac{3}{4}x + 5, \\ \ell_3: & 3x + 4y = 1. \end{cases}$
    (a) Which two lines are parallel? Justify.
    (b) Which two lines are perpendicular? Justify.

11. Solve the system and state your conclusion. $\begin{cases} 4x + 3y = 5 \\ 8x + 5y = 11 \end{cases}$

12. Simplify each of the following. *Do not give decimal answers.*
    (a) $2\sqrt{75}-\sqrt{48}$ (b) $(\sqrt{3}-2)(\sqrt{3}+2)$
    (c) $(\sqrt{5}+\sqrt{2})(2\sqrt{5}-3\sqrt{2})$ (d) $\sqrt{25x^5y^8}$ where $x \geqslant 0$ and $y \geqslant 0$

13. Rationalize the denominator and simplify your answer. $\frac{4}{3+\sqrt{7}}$

14. Rationalize the numerator and simplify your answer. $\frac{\sqrt{x}-2}{x-4}$

15. Use your calculator with 4 decimal places to find the following:
    (a) $\sin 65°$ (b) $\csc 21°$
    (c) $\tan \frac{5\pi}{4}$ (d) $\sec \pi$

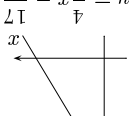
16. $\triangle ABC$ has $\angle C = 90°$ with side $c = 17$ and side $a = 15$. Find side b and $\angle A$.

17. $\triangle ABC$ has $\angle C = 90°$ with side $c = 12$ and $\angle A = 44°$. Find side a.

18. $\triangle ABC$ has $\angle A = 77°$, $\angle B = 38°$ and side $c = 15$. Find side a. (Use the law of sines.)

19. $\triangle ABC$ has $\angle A = 120°$, side $b = 2$ and side $c = 5$. Find side a. (Use the law of cosines.)

ANSWERS

1. (a) $\frac{7}{25}$ (b) 1
2. (a) $\frac{x^2}{yz}$ (b) $\frac{24x^7}{y^2}$
3. (a) $6y^2-14y+5$ (b) $3x^2-x-2$
   (c) $2x^3-9x^2+9x+7$ (d) $x^2-4x+3+\frac{3}{x+2}$
4. (a) $(x-5)(x-2)$ (b) $(2x-y)(4x+3t)$
   (c) $(3x-1)(x+3)$ (d) $(2x-3y)(x-y)$
   (e) $x(x+3)(x-3)$
5. (a) $\frac{1}{y}$ (b) $\frac{-1}{w+3}$ (c) $\frac{3x-5}{(x-2)(x+1)(x-1)}$
   (d) $\frac{3x^2-4x+3}{x^2(x-1)}$
6. $x=-2$ (b) many solutions (c) $x=5$ (d) $x=-5, 6$ (e) $\frac{1\pm\sqrt{5}}{2}$ (f) no real solutions (g) $x=-\frac{2}{3}, 2$ (h) $x=1$
7. (a) $x=2$ (b) \$6048 (c) 8
8. 
   $5x+3y=15$
   x-intercept: $(3,0)$
   y-intercept: $(0,5)$
   Slope: $-\frac{5}{3}$
9. (a) $y=\frac{4}{3}x-\frac{17}{3}$ (b) $x-4y-7=0$
10. (a) $\ell_1 \parallel \ell_3$ (b) $\ell_1 \perp \ell_2$ or $\ell_3 \perp \ell_2$
11. $(2,-1)$
12. (a) $6\sqrt{3}$ (b) -1 (c) $4-\sqrt{10}$ (d) $5x^2\sqrt{xy}^4$
13. $2(3-\sqrt{7})$
14. $\frac{1}{\sqrt{x}+2}$
15. (a) $\sin 65° \approx .9063$ (b) $\csc 21° \approx 2.7904$
    (c) $\tan\frac{5\pi}{4} = 1$ (d) $\sec\pi = -1$
16. $b=8$, $\angle A \approx 61.9275°$
17. $a \approx 8.3359$
18. $a \approx 16.1264$
19. $a \approx 6.2449$