Suppose also that $(\beta_{AB}1_{\kappa_{AB}})\cdot(1_{\kappa_{AB}}\alpha_{AB}) = 1_{\kappa_{AB}}$, and $(1_{\lambda_{AB}}\beta_{AB})\cdot(\alpha_{AB}1_{\lambda_{AB}}) = 1_{\lambda_{AB}}$, ("triangle equalities"). (And if desired, suppose that the "usual coherence conditions" hold.) Then we say $F \underset{R}{\dashv} G$ is a Rax adjunction.

DEFINITION 2. If we reverse the directions of the natural transformations k_{fB} , $k_{A'g}$, ℓ_{fB} , $\ell_{A'g}$, α_{AB} , β_{AB} in Definition 1 , and replace the triangle equalities with $(\kappa\alpha)\cdot(\beta\kappa) = \kappa$, $(\alpha\lambda)\cdot(\lambda\beta) = \lambda$ then $F \underset{L}{\dashv} G$ is a Lax adjunction. (We modify the "usual coherence conditions" suitably, of course.)

PROPOSITION $\mathrm{II}\times\mathrm{II} \begin{array}{c} \xrightarrow{\vee} \\ \xleftarrow[\Delta]{} \\ \xrightarrow[\&]{} \end{array} \mathrm{II}$

are lax 2-functors, as above. Furthermore,

$\vee \underset{R}{\dashv} \Delta$, $\Delta \underset{L}{\dashv} \&$.

REMARK Analogously, $\exists$ (and equality, if we wish to include it) is also a Rax left adjoint to the suitable "diagonal", and $\forall$ is a Lax right adjoint. However, $\supset$ has some of the properties of both types of weak adjunction; this is discussed in [S2] .

The core of the proof is given by the following table:

	$\vee$	$\&$	$\supset$
ι^F	$\vee$-expansion	identity	&-expansion
ι^G	identity	&-expansion	$\supset$-expansion
γ^F	$\vee$-perm. + $\vee$-red.	identity	&-reduction
γ^G	identity	&-reduction	$\supset$-reduction
κ	$\vee$I	&I	&I + $\supset$I
λ	$\vee$E	&E	&E + $\supset$E
$k_{A'g}$	identity	&-reduction	$\supset$-reduction
ℓ_{fB}	$\vee$-perm. + $\vee$-red.	identity	&-reduction
$\ell_{A'g}$	$\vee$-permutation	&-reduction	$\supset$-reduction
k_{fB}	$\vee$-reduction	identity	&-reduction
α	$\vee$-expansion	&-reduction	-
β	$\vee$-reduction	&-expansion	-

The triangle equalities are precisely the principle (R) , (and the "coherence