WEAK ADJOINTNESS IN PROOF THEORY

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0. The idea that (equivalence classes of) derivations in formal logical systems could be considered as morphisms in a category (and *vice versa*) goes back to Lambek [L1] ; the need for an equivalence relation on derivations arises partly so that one has a category, but mainly because one wishes to have the evident correspondences, for example between conjunction and product, implication and exponentiation, and so on. From the proof theorist's point of view, however, this equivalence relation originally appeared somewhat unnatural and *ad-hoc*; furthermore it became quite complicated and unwieldy as one dealt with larger fragments of first order predicate calculus, particularly when working with a sequent calculus. (See, e.g., Szabo [Sz] .) However, when formulated in natural deduction, first order logic has a canonical 2-categorical structure with derivations as morphisms, and although conjunction is not a product, it is a "weak product" in a certain sense; it is the purpose of this note to sketch the precise sense of "weak". This analysis of the 2-categorical structure also motivates and simplifies the description of the equivalence relation on derivations mentioned above: we make the 2-category an ordinary category by making all 2-cells identities, and in the process change a "weak product" into an ordinary product.

For brevity, we shall deal with the natural deduction formulation Π of intuitionistic logic given by Prawitz [P1] . For categorical reasons, it is perhaps desirable to modify Π so that it is multisorted, and so that it allows the interpretation of sorts by empty domains; we leave this and other details to the reader, (they may be found in [S2]).

1. We suppose we are given a language L containing variables, function and predicate symbols; from this we form the system Π of [P1] using the inference rules of natural deduction. There are canonical operations on derivations in Π , given by the reductions and expansions for $\&$, $\vee$, $\supset$, $\forall$, $\exists$, of [P1] . In addition, we will require the following generalisations of $\vee$E-reduction and $\exists$E-reduction: