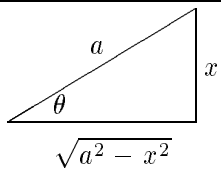
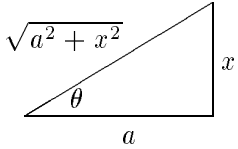
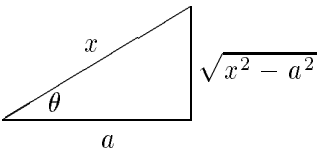




### Trig Substitution

Note: In this table,  $a$  and  $a^2$  are positive constants.

Expression	Triangle	Substitution	Useful Formulas
$a^2 - x^2$		$x = a \sin \theta$ $dx = a \cos \theta d\theta$	$\sqrt{a^2 - x^2} = a \cos \theta$ $a^2 - x^2 = a^2 \cos^2 \theta$
$a^2 + x^2$		$x = a \tan \theta$ $dx = a \sec^2 \theta d\theta$	$\sqrt{a^2 + x^2} = a \sec \theta$ $a^2 + x^2 = a^2 \sec^2 \theta$
$x^2 - a^2$		$x = a \sec \theta$ $dx = a \sec \theta \tan \theta d\theta$	$\sqrt{x^2 - a^2} = a \tan \theta$ $x^2 - a^2 = a^2 \tan^2 \theta$

For general quadratics  $ax^2 + bx + c$ , first complete the square, to reduce to one of the above.

Try these:

1.  $\int \frac{\sqrt{4 - x^2}}{x} dx$
2.  $\int \frac{x dx}{\sqrt{x^2 - 4}}$
3.  $\int \frac{x + 3}{x^2 + 2x + 5} dx$  (\*)
4.  $\int \frac{dx}{\sqrt{6 - 4x - 2x^2}}$  (\*)
5.  $\int \sqrt{9 - 8x^2} dx$
6.  $\int (a^2 - x^2)^{3/2} dx$
7.  $\int \frac{dx}{\sqrt{x^2 - 9}}$
8.  $\int \frac{dx}{\sqrt{4x - 4x^2}}$  (\*)
9.  $\int \frac{\sqrt{x^2 - 9}}{x} dx$
10.  $\int \frac{dx}{\sqrt{x^2 + 6x + 13}}$  (\*)
11.  $\int \frac{x^4 + 1}{x^2 + 1} dx$
12.  $\int \frac{x^3 dx}{(6x^2 + 12x - 5)^{3/2}}$  (\*\*)

(\*) and (\*\*) indicate problems of greater difficulty. You should be able to do the ones marked (\*). The last is a somewhat longer than the others, and is optional.