



Trig Integrals

Type	Substitution	Formulas
$\int \cos^{\text{odd}} x \sin^{\alpha} x dx$ $\int \sin^{\text{odd}} x \cos^{\alpha} x dx$	$u = \sin x \quad du = \cos x dx$ “co-version”: <i>i.e.</i> $u = \cos x \quad du = -\sin x dx$	$\cos^2 x = 1 - \sin^2 x$ $\sin^2 x = 1 - \cos^2 x$
$\int \sin^{\text{even}} x \cos^{\text{even}} x dx$	reduce to lower powers with doubled angles	$\sin^2 x = \frac{1-\cos 2x}{2}$ $\cos^2 x = \frac{1+\cos 2x}{2}$
$\int \sec^{\text{even}} x \tan^{\alpha} x dx$ $\int \csc^{\text{even}} x \cot^{\alpha} x dx$	$u = \tan x \quad du = \sec^2 x dx$ “co-version” $u = \cot x \dots$	$\sec^2 x = 1 + \tan^2 x$ $\csc^2 x = 1 + \cot^2 x$
$\int \tan^{\text{odd}} x \sec^{\alpha} x dx$ $\int \cot^{\text{odd}} x \csc^{\alpha} x dx$	$u = \sec x \quad du = \sec x \tan x dx$ “co-version” $u = \csc x \dots$	$\tan^2 x = \sec^2 x - 1$ $\cot^2 x = \csc^2 x - 1$
$\int \tan^{\text{even}} x dx$ $\int \cot^{\text{even}} x dx$	convert to $\sec x$ convert to $\csc x$	$\tan^2 x = \sec^2 x - 1$ $\cot^2 x = \csc^2 x - 1$
$\int \sec^{\text{odd}} x dx$ $\int \csc^{\text{odd}} x dx$ ($\int \sec^{\text{odd}} x \tan^{\text{even}} x dx, \int \csc^{\text{odd}} x \cot^{\text{even}} x dx$ handled the same way—convert the tan to sec, <i>etc.</i>)	integration by parts, recursion “co-version”	$u = \sec^{\text{odd}-2} x \quad dv = \sec^2 x dx$
Anything else	convert to sin and cos (and be inventive!)	

Try these:

1. $\int \sin^3 x dx$
2. $\int \sin^4 x dx$
3. $\int \cos^4 x \sin^2 x dx$
4. $\int \tan^5 x dx$
5. $\int \sqrt{\tan x} \sec^2 x dx$
6. $\int \sqrt{\sec x} \tan^3 x dx$
7. $\int \sec^4 x \tan^3 x dx$
8. $\int (\csc x)^{2/3} \cot^3 x dx$
9. $\int \cot^3 x dx$
10. $\int \sqrt[3]{\cot x} \csc^4 x dx$
11. $\int \frac{\tan(\ln x) dx}{x}$
12. $\int \sin^3 6x dx$
13. $\int x \sqrt{\sec(x^2)} \tan^3(x^2) dx$