Instructor: Dr. R.A.G. Seely

Test 3
(version A)

Cal II (S) (Maths 201–NYB)

(Marks)

Remember that the use of any calculator is not permitted. Please show all your work, so as to justify your answers. *Answers without justification will not receive full credit.* Presentation is important, and some credit will be lost for messy or incoherent work.

(2×2) 1. For each of the following sequences, does the sequence converge? And if so, find its limit as $n \to \infty$.

(a) $\left\{ n \sin\left(\frac{1}{n}\right) \right\}$ (b) $\left\{ n - \sqrt{n} \right\}$

(2×3) 2. For each of the following series, determine whether or not it converges, and if it does, find the sum of the series.

(a) $\displaystyle\sum_{n=1}^{\infty} \left(1 - \frac{3}{n}\right)^n$ (b) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n^2} - \frac{1}{(n+1)^2}\right)$

(16) 3. Determine whether each of the following series converges or diverges. State the tests you use, and verify that the conditions for using them are satisfied.

(a) $\displaystyle\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^2}\right)$ (b) $\displaystyle\sum_{n=0}^{\infty} \frac{(2n)!}{2^n\, n!}$

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{\ln n}{n^2}$ (d) $\displaystyle\sum_{n=1}^{\infty} \frac{5^{2n}}{n^n}$

(10) 4. Classify each of the following series as absolutely convergent, conditionally convergent or divergent. (Justify your conclusions.)

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2^n}{n^2}$ (b) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{n+1}{\sqrt[4]{2n^9 + 6n + 1}}$

(5) 5. Determine the interval of convergence of the series $\displaystyle\sum_{n=0}^{\infty} \frac{(x+2)^n}{4^n \sqrt{n^2+1}}$.

(2×2) 6. Suppose that $\displaystyle\sum_{n=0}^{\infty} a_n$ converges, $a_n \geq 0$ for all $n \geq 0$.

(a) What is $\displaystyle\lim_{n\to\infty} a_n$? (b) Does $\displaystyle\sum_{n=0}^{\infty} \frac{n\, a_n}{2n+1}$ converge?

Be sure to (briefly!) justify your answers (*e.g.* mention which theorem or convergence criterion you are using).

(5) 7. Find the Maclaurin series for $f(x) = \dfrac{x}{x+1}$. Write down the first four non-zero terms explicitly, and give a general formula for the series. What is the interval of convergence for this series?

(Total: 50)