Calculus II (Maths 201–NYB)

1. Find the sum of $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2+k}$ if it converges.

2. Converge or diverge?:

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+1}{\sqrt{n^5+2n^2}}$ (b) $\displaystyle\sum_{k=1}^{\infty} \frac{k}{e^{k^2}}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2\,5^n}{(2n)!}$

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{\sec^2(n)}{\sqrt[3]{n}}$ (e) $\displaystyle\sum_{k=1}^{\infty} \left(1-\frac{1}{2k}\right)^k$ (f) $\displaystyle\sum_{n=0}^{\infty} \frac{2+3^n}{5^n}$

3. Converge absolutely, conditionally, or diverge?:

   (a) $\displaystyle\sum_{k=2}^{\infty} \frac{(-1)^k}{k\sqrt{\ln k}}$ (b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n\,n^2}{\sqrt[3]{2n^4+n+1}}$ (c) $\displaystyle\sum_{k=0}^{\infty} (-1)^k\,\frac{k!}{(2k)!}$

4. Find the interval of convergence of $\displaystyle\sum_{n=1}^{\infty} \frac{(x+1)^n\,\sqrt[3]{n}}{5^n}$.

5. Find the Maclaurin series for $f(x) = (x+1)\ln(x+1)$. Write the first 6 non-zero terms explicitly, and express the n^{th} term in terms of a general formula. Write the series in sigma notation. What is the interval of convergence?

Answers:

1. 1

2. D (LCT with $\sum \frac{1}{n}$) C (RT or JT) C (RT) D (CT) D (nTT) C (GS)

3. CC (JT & AST) D (nTT) AC (RT)

4. $(-6, 4)$ (RT; nTT at endpoints)

5. $x + \displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{n+1}}{n(n+1)}$