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Test 3

(version for practice)

Cal I (S) (Maths 201–NYA)

1. For a continuous function $y = f(x)$, suppose you know that its derivative is postive for $-\infty < x < 1$ and for $3 < x < +\infty$, and that its derivative is negative for $1 < x < 3$. Furthermore suppose you know that its second derivative is positive for $-\infty < x < 0$ and $2 < x < 4$, and is negative for $0 < x < 2$ and $4 < x < +\infty$. Finally, suppose you know the x-axis is a horizontal asymptote and that $(2, 0)$ is the only x-intercept. Draw a rough sketch of the function showing all these properties.

2. For each of the following functions, graph the function, identifying all intercepts, aymptotes, local extrema, and inflection points. Specify intervals where the graph is increasing, decreasing, concave up, and concave down. Show all your work. In the case of the second function, the first and second derivatives are already calculated for you.

   (a) $f(x) = \frac{3}{4}x^4 - 6x^2 + 12$

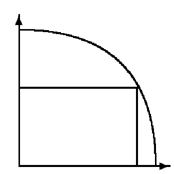
   (b) $f(x) = \dfrac{x+1}{(x-1)^2} \qquad f'(x) = -\dfrac{x+3}{(x-1)^3} \qquad f''(x) = \dfrac{2(x+5)}{(x-1)^4}$

   (c) $y = 1 + \dfrac{2}{x} + \dfrac{1}{x^2}$

3. For the function $f(x) = x + \sqrt{1-x}$, defined for $x \leq 1$, determine whether or not it has a global maximum and whether or not it has a global minimum. Justify your answer by showing all necessary calculations.

4. Find the values of x where the minimum and maximum values of the function $f(x) = x\sqrt{1+x}$ occur on the interval $[-1, 1]$.

5. A rectangle is to be inscribed in a quarter circle of radius $= 1$ as shown in the figure; find the dimensions that give the rectangle with the largest possible area.



6. Evaluate the following:

   (a) $\displaystyle\int \frac{3x^4 - 5x + 2}{\sqrt[3]{x}}\,dx$ (b) $\displaystyle\int_1^2 \left(x - \frac{1}{x}\right)^2 dx$

   (c) $\displaystyle\int_0^{\frac{\pi}{6}} \left(t^2 - \frac{1}{\cos^2 t}\right) dt$ (d) $\displaystyle\int \left(\mathrm{e}^2 + \sqrt[5]{x^3} - \mathrm{e}^x\right) dx$

7. Given $f''(x) = 2 + \sin x$ satisfying $f'(\pi) = 2\pi,\ f(\pi) = 0$, find the function f.

8. What is the minimum value of the function $f(x) = \displaystyle\int_0^{x^2} \frac{1}{1+t^2}\,dt$?

9. The graph of $y = \frac{3}{4}x^4 - 6x^2 + 12$ (see question 2(a) above) has two x-intercepts (at ± 2), and so there is a closed region between them, the curve, and the x-axis. Find the area of that region.