Volumes of Revolution

Determine the volume of the solid generated when the area bounded by the given curves is rotated about the stated axis.

1. $y = 4 - x^2$, x-axis, y-axis, quadrant I; about the x-axis.
2. $y = x^3$, $y = x$: about the y-axis.
3. $y = \sin x$, x-axis, $x = 0$, $x = \pi$; about the x-axis. Set up the integral, but do not evaluate.
4. $y = 2x - x^2, y = x^2$; about the x-axis. Set up the integral, but do not evaluate.
5. $y = x - 2, y = -\frac{1}{2}(x-2)^2$; about x-axis. Set up the integral, but do not evaluate.
6. $y = \cos x, y = \sin x, x = 0, x = \frac{\pi}{2}$, about the x-axis. Set up the integral, but do not evaluate.
7. $y = \arcsin x, y = -\frac{\pi}{2}, y = \frac{\pi}{2}$, y-axis; about the y-axis. Set up the integral, but do not evaluate.
8. $y = x^2, x = y^2$ about the x-axis using the shell method. Set up the integral, but do not evaluate.
9. $y = x^2, x = y^2$ about the x-axis using the disk method. Set up the integral, but do not evaluate.
10. $x = y^2 - 2y, x - 2y + 3 = 0$; about the x-axis. Set up the integral, but do not evaluate.
11. $y = \dfrac{1}{\sqrt{1+x^2}}, y = 1, x = 2$; about the y-axis. Set up the integral, but do not evaluate.
12. $y = \dfrac{1}{x^2 - 4x}, y = 0, x = 5, x = 10$; about the x-axis. Set up the integral, but do not evaluate.
13. $y = \sin x$, x-axis, $x = 0, x = \pi$; about the y-axis. Set up the integral, but do not evaluate.
14. $y = \begin{cases} \frac{3}{4} & 0 \le x \le 1 \\ \frac{3}{4}x^2 & 1 \le x \le 2 \end{cases}$ in the first quadrant about the x-axis.
15. $y = e^x, y = e$, y-axis; about the x-axis.
16. $y = e^x, y = e$, y-axis; about the y-axis.
17. $y = x^2, y = 1, x = 3$; about the x-axis using the disk method.
18. $y = x^2, y = 1, x = 3$; about the x-axis using the shell method.
19. $y = x^2 + 2, y = x + 8$; about the x-axis.
20. $y = 4x, y = 2x^2$; about the y-axis.
21. $y = x^3(2 - x)$, x-axis, first quadrant; about $x = 0$.
22. $f(x) = 5x, g(x) = x^2, x = 0, x = 3$; about the x-axis.
23. $f(x) = (x-2)^2$ for $1 \le x \le 3$, x-axis; about the y-axis.
24. Set up, but do not evaluate, an integral for the volume of revolution obtained when the region bounded by $y = \arctan x$, x-axis, $x = 1$, is revolved about the y-axis.
25. Set up, but do not evaluate, an integral for the volume of revolution obtained when the region bounded by $y = \sqrt{x}, x = 1, x = 4, y = 0$ is revolved about the y-axis.
26. Set up, but do not evaluate, an integral for the volume of revolution obtained when the region bounded by $f(x) = x^2 + 1, g(x) = 2x + 4, x = 0$, in the first quadrant, is revolved about the y-axis.

Answers:

1. $\int_0^2 \pi(4-x^2)^2\,dx = \frac{256\pi}{15}$ [disks]
2. $\int_0^1 2\pi x(x-x^3)\,dx + \int_{-1}^0 2\pi(-x)(x^3-x)\,dx = \frac{8\pi}{15}$ [shells]
3. $\int_0^\pi \pi\sin^2 x\,dx$ [disks]
4. $\int_0^1 \pi[(2x-x^2)^2-(x^2)^2]\,dx$ [disks]
5. $\int_0^2 \pi\left[(x-2)^2-\frac{1}{4}(x-2)^4\right]\,dx$ [disks]
6. $\int_0^{\pi/4} \pi(\cos^2 x-\sin^2 x)\,dx + \int_{\pi/4}^{\pi/2} \pi(\sin^2 x-\cos^2 x)\,dx$ [disks]
7. $\int_{-\pi/2}^0 \pi(-\sin y)^2\,dy + \int_0^{\pi/2} \pi(\sin y)^2\,dy$ [disks]
   or $\int_{-1}^0 2\pi(-x)[\arcsin x-(-\frac{\pi}{2})]\,dx + \int_0^1 2\pi x[\frac{\pi}{2}-\arcsin x]\,dx$ [shells]
8. $\int_0^1 2\pi y(\sqrt{y}-y^2)\,dy$ [shells]
9. $\int_0^1 \pi\left[(\sqrt{x})^2-(x^2)^2\right]\,dx$ [disks]
10. $\int_1^3 2\pi y\left[(2y-3)-(y^2-2y)\right]\,dy$ [shells]
11. $\int_0^2 2\pi x\left[1-\frac{1}{\sqrt{1+x^2}}\right]\,dx$ [shells]
12. $\int_5^{10} \pi\left(\frac{1}{x^2-4x}\right)^2\,dx$ [disks]
13. $\int_0^\pi 2\pi x\sin x\,dx$ [shells]
14. $\int_0^1 \pi\left(\frac{3}{4}\right)^2\,dx + \int_1^2 \pi\left(\frac{3x^2}{4}\right)^2\,dx = \frac{81\pi}{20}$ [disks]
15. $\int_0^1 \pi(e^2-e^{2x})\,dx = \frac{\pi}{2}(e^2+1)$ [disks]
16. $\int_0^1 2\pi x(e-e^x)\,dx = \pi(e-2)$ [shells]
17. $\int_1^3 \pi\left[(x^2)^2-1^2\right]\,dx = \frac{232\pi}{5}$ [disks]
18. $\int_1^9 2\pi y(3-\sqrt{y})\,dy = \frac{232\pi}{5}$ [shells]
19. $\int_{-2}^3 \pi\left[(x+8)^2-(x^2+2)^2\right]\,dx = 250\pi$ [disks]

20. $\int_0^2 2\pi x[4x - 2x^2]\,dx = \frac{16\pi}{3}$ [shells]

21. $\int_0^2 2\pi x[x^3(2 - x)]\,dx = \frac{64\pi}{15}$ [shells]

22. $\int_0^3 \pi\left[(5x)^2 - (x^2)^2\right]\,dx = \frac{882\pi}{5}$ [disks]

23. $\int_1^3 2\pi x(x - 2)^2\,dx = \frac{8}{3}\pi$ [shells]

24. $\int_0^1 2\pi x \arctan x\,dx$ [shells]

25. $\int_1^4 2\pi x\sqrt{x}\,dx$ [shells]

26. $\int_0^3 2\pi x\left[(2x + 4) - (x^2 + 1)\right]\,dx$ [shells]

From the files of Norman Dobson
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