1 Tangents and Normals

1. Find equations of (a) the tangent line and (b) the normal line to $y = \dfrac{1}{x-1}$ at $(-2, -\frac{1}{3})$

2. Find the slope of the tangent line to $xy^4 - \dfrac{x}{y} = 1$ at $(\frac{2}{31}, 2)$

3. Find the coordinates of the point(s) on the curve $y = 2x^3 - 2x + 4$ where the tangent lines are parallel to the line $y = 22x - 9$.

4. Find the equation of the tangent line to $x^3 + x^2y + y^2 - x = 0$ at $(1, -1)$.

5. Find equations of (a) the tangent line and (b) the normal line to $y = \dfrac{x}{x+4}$ at $(4, \frac{1}{2})$.

6. Find the equation of the tangent line to $x^2 + 3xy + y^2 = 5$ at $(1, 1)$.

7. Find the slope of the tangent line to the curve $y = x^2$ at $(-1, 1)$.

8. Find the equation of the tangent line to $x^2y + xy^2 = 6$ at $(2, 1)$.

9. Find the equation of the normal line to $x^2y + y^2 - 4x + 6y = 16$ at $(2, 2)$.

10. Find the equation of the tangent line to $y = (x^2 - 2)^8(3x - 2)^7$ at $x = 1$.

11. Find the slope of the tangent to $3xy + y^2 = 5x + 17$ at $(2, 3)$.

12. Find the slope of the tangent line to $x^2 + 2xy + 2y^2 = 10$ at $(-2, 3)$.

13. Find the slope of the tangent line to $xy^2 - 2x^3 = 2$ at $(2, -3)$.

14. Find an equation of the tangent line to the graph of $f(x) = \sqrt{x^2 + 3}$ at the point where $x = 1$.

15. Find the equation of the line tangent to the graph of $y = \dfrac{2x+1}{3x-1}$ at the point where $x = 1$.

16. Given the curve $x^2 + y^2 - \ln x^2 + \ln y = 2$

    (a) find $\dfrac{dy}{dx}$ at $(-1, 1)$,

    (b) find an equation of the normal to the curve at the point $(-1, 1)$, and

    (c) find all values of x for which the tangent line is horizontal.

17. Find all values of x where the tangent to $y = 2x^3 + 9x^2 + 5$ has a slope of 24.

18. Find the equation of the tangent line to the curve $x^2y - y^3 = 8$ at the point $(-3, 1)$.

19. Find the slope of the tangent line to the curve $4y^3 - x^2y + x = 2$ at the point $(2, -1)$.

20. Find all points on the curve $y = x^3 + 2x^2 - 6x + 5$ where the tangent is parallel to the line $2x + y = 4$.

21. Find the equations of the tangent and normal lines to $3x^2y + 5x + \sqrt{y} = 19$ at $(1, 4)$.

22. Find the equation of the line tangent to the curve $x^3 - 2xy + y^4 = 8$ at $(2, 0)$.

23. Find the equation for the tangent line to the curve $y = \ln(3x^2 - 11) - 5x$ at $(2, -10)$.

Answers:

1. (a) $9y + x + 5 = 0$ (b) $3y - 27x - 53 = 0$
2. $-\frac{961}{129}$
3. $(2, 16)$ and $(-2, -8)$
4. $y = -1$
5. (a) $16y - x - 4 = 0$ (b) $2y + 32x - 129 = 0$
6. $y + x - 2 = 0$
7. -2
8. $8y + 5x - 18 = 0$
9. $2y - 7x + 10 = 0$
10. $y - 5x + 4 = 0$
11. $3y + x - 11 = 0$
12. $4y + x - 10 = 0$
13. $-\frac{5}{4}$
14. $2y - x - 3 = 0$
15. $4y + 5x - 11 = 0$
16. (a) 0 (b) $x = -1$
    (c) $x = 1$ and $x = -1$
17. $x = 1$ and $x = -4$
18. $y - x - 4 = 0$

19. $-\frac{5}{8}$

20. $(\frac{2}{3}, \frac{59}{27})$ and $(-2, 17)$

21. tangent: $13y + 116x - 168 = 0$ and
    normal: $116y - 13x - 451 = 0$

22. $y - 3x + 6 = 0$

23. $y = 7x - 24$