1 Related Rates

1. Water is pumped into a trough 8 feet long with a triangular cross-section 2 feet by 2 feet by 2 feet at the rate of 16 cubic feet^3 per minute. How fast is the water level rising when the water is 1 foot deep?

2. The area of a circle is 10 square inches and is increasing at the rate of 4 inches per minute. At what rate is the radius increasing?

3. A balloon expands so that after t seconds its radius is given by $r = t^2 + 2t - 1$ inches.

   (a) Find the rate of change of the radius at $t = 2$ seconds.

   (b) Find the rate of change of the volume at $t = 2$ seconds.

4. The volume of a sphere is 4 cubic feet and is increasing at the rate of 12 cubic feet per minute. At what rate is the radius increasing? (Volume of a sphere: $V = \frac{4}{3}\pi r^3$)

5. The area of a square is increasing at the rate of 6 square inches per minute. At what rate is a side increasing when the area is 10 square inches?

6. A stone thrown into a still pond produces concentric circular ripples. If the radius of the largest ripple increases at the rate of 0.60 meters per second, how fast is the area within this ripple increasing when the radius is 50 meters?

7. The volume of a cube is decreasing at the rate of 2 cubic centimeters per hour. Find the rate at which the surface area is decreasing at the time when the volume is 343 cubic centimeters.

8. Assume that a tree trunk has a conical shape, with radius r changing at the rate of $\frac{1}{10}$ meter per year. The height of the tree changes at the rate of 2 meters per year. Compute the rate of change of the volume when the tree is 50 meters tall and has a radius of 1 meter. (Volume of a cone: $V = \frac{1}{3}\pi r^2 h$)

9. Atmospheric pressure at altitude h feet above sea level is given by

   $$p = 15e^{-0.004h} \text{ pounds/square inch}$$

   A jet liner at 10,000 feet is climbing at the rate of 1000 feet per minute. Find the rate of change of external air pressure at 10,000 feet.

10. A fuel tank in the shape of an inverted right circular cone is being filled at the rate of 2 cubic meters per minute. The height of the cone is 16 meters and the radius 4 meters. How fast is the fuel level rising when the fuel is 5 meters deep?

11. One bicycle is east of an intersection, and it is travelling toward the intersection at the rate of 9 miles per hour. At the same time a second bicycle is south of the intersection, and it is travelling away from the intersection at the rate of 10 miles per hour. Is the distance between the bicycles increasing or decreasing when the first is 4 miles east and the second is 4 miles south of the intersection? At what rate?

12. A ladder 20 meters long is lying against a building. The top of the ladder begins to slide down the wall. How fast is the distance of the top of the ladder from the ground changing when the angle φ between the ladder and the ground is 30° and φ is changing at the rate of 2 radians per second?

13. A kite 60 meters above the ground is moving horizontally at 3 meters per second. At what rate is the length of the string changing when 100 meters of string is paid out? (Assume the string forms a straight line.)

14. A spherical balloon, initially inflated to a volume of over 30,000 cubic meters, springs a leak which causes the radius to decrease at a rate of 2 meters per minute. At the instant when the radius is 3 meters, how fast (in cubic meters per minute) is the hot air escaping from the balloon?

15. The law of cosines, which relates the lengths of the three sides of any triangle, is $c^2 = a^2 + b^2 - 2ab\cos C$, where C is the angle opposite side c. If side a is fixed at 1 centimeter and side b is fixed at 2 centimeters, and if the angle C is increasing at the constant rate of 0.2 radians per minute, find the rate at which side c is increasing at the instant when $c = \sqrt{3}$ centimeters.

16. An aircraft is flying horizontally at a rate of 13 miles per minute at an altitude of 5 miles.

It passes over a radio beacon at exactly 3:00 PM. How fast is the distance between the aircraft and the beacon increasing exactly one minute later? (The beacon is assumed to be at ground level.)

17. A monkey climbs a vertical pole, pulling behind it a long rope which runs through a ring at ground level 20 feet from the base of the pole. If the rate of climbing is 2 feet per second, at what rate is the rope running through the ring when the monkey is 15 feet above the ground? (Assume the rope does not sag.)

18. A ladder 30 feet long is leaning against a vertical wall when the top of the ladder starts to slide down the wall at the rate of 3 feet per second. At what rate is the bottom of the ladder moving away from the wall when the top of the ladder is 12 feet from the ground?

19. A ladder of length 27 feet that is leaning against a wall has its upper end sliding down the wall at a rate of $\frac{1}{6}$ foot per second. What is the rate of change of the measure of the acute angle made by the ladder with the ground when the upper end is 6 feet above the ground?

20. Two concentric circles are expanding. The radius of the larger circle is increasing at a constant rate of 2 centimeters per minute, while that of the smaller is increasing at a constant rate of 3 centimeters per minute. At the instant when the radius of the larger circle is 10 centimeters and that of the smaller is 7 centimeters, at what rate is the area between the circles changing? Is this increasing or decreasing?

21. A searchlight is trained on an aircraft that flies directly above the light at a constant altitude of 2 kilometers and speed of 0.2 kilometers per second. Two seconds after the plane is directly over the light, how fast is the light rotating?

22. Show that the rate of change of the radius of a circle with respect to its circumference is independent of the radius.

23. The total resistance R of two parallel resistors, R_1 and R_2, is given by:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

If R_1 is increasing at 0.12 ohms per second, while R_2 is decreasing by 0.08 ohms per second find the rate of change of the total resistance when R_1 is 4 ohms and R_2 is 2 ohms. Is the total resistance increasing or decreasing at this time?

24. Sand is being poured onto the ground forming a conical pile with height equal to one-fourth the diameter of the base. If the sand is falling at a rate of 20 cubic centimeters per second, how fast is the height increasing when it is 3 centimeters?

25. An observer is located on the ground 6 kilometers from the point where a space shuttle is launched (vertically). When the shuttle is 8 kilometers high, it is travelling at 500 kilometers per hour. Determine, for that moment, the rate at which the distance between the observer and the shuttle is changing.

Answers:

1. $\sqrt{3}$ feet per minute
2. $\frac{2}{\sqrt{10\pi}}$ inches per minute
3. (i) 6 inches per second (ii) 1176π cubic inches per second
4. $\sqrt[3]{\frac{3}{\pi}}$ feet per minute
5. $\frac{3}{\sqrt{10}}$ inches per minute
6. 60π square meters per second
7. $\frac{8}{7}$ square centimeters per hour
8. 4π cubic meters per year
9. $-60e^{-40}$ pounds per square inch per minute
10. $\frac{32}{25\pi}$ meters per minute
11. Increasing at $\frac{1}{\sqrt{2}}$ miles per hour
12. $20\sqrt{3}$ meters per second
13. $\frac{12}{5}$ meters per second
14. 72π cubic meters per minute
15. 0.2 centimeters per minute
16. $\frac{169}{\sqrt{194}}$ kilometers per minute
17. $\frac{6}{5}$ feet per second
18. $\frac{6}{\sqrt{21}}$ feet per second

19. $-\frac{1}{6\sqrt{693}}$ radians per second
20. Decreasing at 2π square centimeters per minute
21. $\frac{5}{52}$ radians per second
22. $\frac{dr}{dC} = \frac{1}{2\pi}$
23. $-{}^{1}\!/_{45}$ ohms per second; Decreasing
24. $\frac{5}{9\pi}$ centimeters per second
25. 400 kilometers per hour