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Calculus II – Final Exam Problems

Positive Series

Determine whether the series converge or diverge.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+3)}$
2. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$
3. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)}}$
4. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{e^n}$
5. $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{(n+1)^3}$
6. $\displaystyle\sum_{n=1}^{\infty} \frac{2n+1}{n3^n}$
7. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[6]{n^4}}$
8. $\displaystyle\sum_{n=1}^{\infty} \frac{4}{n(n+1)(n+2)}$
9. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{3^n}$
10. $\displaystyle\sum_{n=1}^{\infty} \frac{(n+1)!}{(2n+1)!}$
11. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+1}{n+5}$
12. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n}$
13. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n!}$
14. $\displaystyle\sum_{n=1}^{\infty} \frac{\pi}{(2n+3)(n-5)}$
15. $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^2 2^n}{(2n)!}$
16. $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n^2+n}}{n^3}$
17. $\displaystyle\sum_{n=2}^{\infty} \frac{n(1+n^2)}{n^3-3}$
18. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{2^n 3^n}$
19. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3+2^{-n}}$
20. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n[\ln n]^{3/2}}$
21. $\displaystyle\sum_{n=1}^{\infty} \frac{5^n+2^n}{4^n}$
22. $\displaystyle\sum_{n=2}^{\infty} \frac{\sqrt{n}}{\ln n}$
23. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{\sqrt{n^8+6n^3}}$
24. $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n+1}{n+2}\right)$
25. $\displaystyle\sum_{n=2}^{\infty} \left(\frac{n}{\ln n}+\frac{3}{4^n}\right)$
26. $\displaystyle\sum_{n=1}^{\infty} (e^{1/n}-1)$
27. $\displaystyle\sum_{n=1}^{\infty} \frac{\arctan n}{1+n^2}$
28. $\displaystyle\sum_{n=1}^{\infty} \frac{n^3+1}{n!}$
29. $\displaystyle\sum_{n=1}^{\infty} \frac{5n}{(n^7+3)^{1/3}}$
30. $\displaystyle\sum_{n=1}^{\infty} \frac{n(n-1)}{(n+1)(n+2)}$
31. $\displaystyle\sum_{n=2}^{\infty} \left(\frac{n}{\ln n}-\frac{5}{2^n}\right)$
32. $\displaystyle\sum_{n=1}^{\infty} \frac{5n-6}{3n^{5/2}+2n-2}$
33. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{2^n n^2}$
34. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$
35. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{(2n)!}$
36. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n+n}$
37. $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{e^n}$
38. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$
39. $\displaystyle\sum_{n=1}^{\infty} \frac{e^{-\sqrt{n}}}{\sqrt{n}}$
40. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n(n+3)}$
41. $\displaystyle\sum_{n=1}^{\infty} \frac{3n^4-2n^3+7}{5n^7+n^2-3n}$
42. $\displaystyle\sum_{n=2}^{\infty} \frac{n+3}{(\ln n)^2}$
43. $\displaystyle\sum_{n=1}^{\infty} \frac{2n-1}{\sqrt{n^5+2}}$
44. $\displaystyle\sum_{n=1}^{\infty} \frac{5n^2}{(n+1)(n+2)(n+3)(n+4)}$
45. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n+1)^{1/2}}$
46. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\ln^2 n}$

47. $\sum_{n=1}^{\infty} \sin n\pi$

48. $\sum_{n=1}^{\infty} \frac{n^2}{n^3 - 1000}$

49. $\sum_{n=1}^{\infty} \frac{1}{n \ln n}$

50. $\sum_{n=1}^{\infty} \frac{n}{n+1}$

51. $\sum_{n=1}^{\infty} \frac{n}{2^n}$

52. $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

53. $\sum_{n=1}^{\infty} \frac{\sqrt{n+2}}{n^3+1}$

54. $\sum_{n=1}^{\infty} \frac{(n+1)!}{(n-1)!2^n}$

55. $\sum_{n=1}^{\infty} \left(1 + \frac{4}{n}\right)^n$

56. $\sum_{n=1}^{\infty} \ln \frac{n}{n+1}$

57. $\sum_{n=1}^{\infty} ne^{-n^2}$

58. $\sum_{n=1}^{\infty} \frac{10n^3 + 7n^2}{n^{3/2} + 3}$

59. $\sum_{n=1}^{\infty} \frac{3^n}{(2n)!}$

60. $\sum_{n=1}^{\infty} \frac{7^n}{(3n)!}$

61. $\sum_{n=1}^{\infty} \frac{1}{3 \cdot 2^n + 5}$

62. $\sum_{n=1}^{\infty} \frac{n+2}{n\sqrt{5n-2}}$

63. $\sum_{n=1}^{\infty} \frac{n^2 - 2n + 1}{(n-3)(n+2)}$

64. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{99+n^2}}$

65. $\sum_{n=1}^{\infty} \frac{1}{(2+\frac{1}{n})^n}$

66. $\sum_{n=1}^{\infty} \frac{e^n}{n^3}$

67. $\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$

68. $\sum_{n=1}^{\infty} \frac{n^2 + 3n + 1}{4n^3 + 1}$

69. $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{3/2}}$

70. $\sum_{n=2}^{\infty} \frac{4}{3^n}$

71. $\sum_{n=1}^{\infty} \frac{e^n}{n!}$

72. $\sum_{n=1}^{\infty} \ln\left(\frac{n+1}{2n+1}\right)$

73. $\sum_{k=1}^{\infty} \frac{k^2}{k!}$

74. $\sum_{n=1}^{\infty} \frac{5n}{(n^7+3)^{1/3}}$

75. $\sum_{k=2}^{\infty} \frac{1}{k\sqrt[3]{\ln k}}$

76. $\sum_{k=1}^{\infty} \frac{\sqrt{k}}{k^2+4}$

77. $\sum_{m=1}^{\infty} \frac{\arctan m}{1+m^2}$

78. $\sum_{n=1}^{\infty} \frac{n+3}{2^n}$

79. $\sum_{n=1}^{\infty} \frac{n+2}{\sqrt{3n^3+n+1}}$

80. $\sum_{n=1}^{\infty} \left\{\frac{1}{2} + \frac{2}{3n}\right\}^n$

81. $\sum_{n=2}^{\infty} \left(\frac{\sqrt{n}}{\ln n} + (-1)^n \frac{1}{3^n}\right)$

82. $\sum_{n=1}^{\infty} \frac{(n+2)!}{4^{n+1}}$

83. $\sum_{n=1}^{\infty} n^2 e^{-n^3}$

84. $\sum_{n=1}^{\infty} \frac{5\sqrt{n^2+2}}{3n+1}$

85. $\sum_{n=1}^{\infty} \frac{2\sqrt{n}+5n}{\sqrt{3n^3+4}}$

86. $\sum_{n=3}^{\infty} \frac{e}{(3n+2)(2n-5)}$

87. $\sum_{m=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2m-1)}{m!}$

88. $\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2}$

89. $\sum_{n=1}^{\infty} \frac{\cos\frac{1}{n}}{\arctan n}$

90. $\sum_{n=1}^{\infty} \frac{3n+2}{(n+1)(4n)}$

91. $\sum_{n=1}^{\infty} \ln\left[\frac{n^2+2}{3n^2+5}\right]$

92. $\sum_{n=1}^{\infty} \frac{e^n}{(2n+1)!}$

93. $\sum_{n=1}^{\infty} \left[\frac{n+3}{2n+1}\right]^n$

94. $\sum_{n=1}^{\infty} \frac{1}{2+5^n}$

95. $\sum_{n=1}^{\infty} ne^{-n}$

96. $\sum_{n=1}^{\infty} \cos(n\pi)$

97. $\sum_{n=1}^{\infty} \frac{(n+1)!}{3^{2n+1}}$

98. $\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} + \frac{1}{5^n}\right)$

99. $\sum_{n=1}^{\infty} \frac{(2n)!}{3^n}$

100. $\sum_{n=1}^{\infty} \frac{3^{2n}}{1+\ln n}$

101. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n^2+3}}$

102. $\displaystyle\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$

103. $\displaystyle\sum_{n=1}^{\infty} \frac{4\sqrt{n}-1}{n^2+2\sqrt{n}}$

104. $\displaystyle\sum_{n=1}^{\infty} \ln\left[\frac{2n+1}{n}\right]$

105. $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n5^n+2}$

106. $\displaystyle\sum_{n=1}^{\infty} n\sin\left(\frac{1}{n}\right)$

Answers

1. Converges (LCT)
2. Diverges (p-series)
3. Diverges (LCT)
4. Converges (Ratio Test)
5. Converges (LCT)
6. Converges (Ratio Test)
7. Diverges (p-series)
8. Converges (LCT)
9. Converges (Ratio Test)
10. Converges (Ratio Test)
11. Diverges (DT)
12. Diverges (IT)
13. Converges (Ratio Test)
14. Converges (LCT)
15. Converges (Ratio Test)
16. Converges (LCT)
17. Diverges (DT)
18. Diverges (Ratio Test)
19. Diverges (DT)
20. Converges (IT)
21. Diverges (geometric series)
22. Diverges (DT)
23. Converges (LCT)
24. Diverges ($\lim S_n$)
25. Diverges (DT)
26. Diverges (LCT)
27. Converges (IT)
28. Converges (Ratio Test)
29. Converges (LCT)
30. Diverges (DT)
31. Diverges (DT)
32. Converges (LCT)
33. Diverges (Ratio Test)
34. Diverges (IT)
35. Converges (Ratio Test)
36. Converges (CT)
37. Converges (Ratio Test)
38. Converges (IT)
39. Converges (IT)
40. Diverges (Ratio Test)
41. Converges (LCT)
42. Diverges (DT)
43. Converges (LCT)
44. Converges (LCT)
45. Diverges (LCT)
46. Converges (IT)
47. Converges ($\lim S_n$)
48. Diverges (LCT)
49. Diverges (IT)
50. Diverges (DT)
51. Converges (Ratio Test)
52. Converges (LCT)
53. Converges (LCT)
54. Converges (Ratio Test)
55. Diverges (DT)
56. Diverges ($\lim S_n$)
57. Converges (IT)
58. Diverges (DT)
59. Converges (Ratio Test)
60. Converges (Ratio Test)
61. Converges (CT)
62. Diverges (LCT)
63. Diverges (DT)
64. Diverges (LCT)
65. Converges (CT)
66. Diverges (Ratio Test)
67. Converges (Ratio Test)
68. Diverges (LCT)
69. Converges (IT)
70. Converges (geometric-series)
71. Converges (Ratio Test)
72. Diverges (DT)
73. Converges (Ratio Test)
74. Converges (LCT)
75. Diverges (IT)
76. Converges (LCT)
77. Converges (IT)
78. Converges (Ratio Test)
79. Diverges (LCT)
80. Converges (Root Test)
81. Diverges (DT + geometric series)

82. Diverges (Ratio Test)
83. Converges (IT)
84. Diverges (DT)
85. Diverges (LCT)
86. Converges (LCT)
87. Diverges (LCT)
88. Converges (IT)
89. Diverges (DT)
90. Diverges (LCT)
91. Diverges (DT)
92. Converges (Ratio Test)
93. Converges (Root Test)
94. Converges (CT)
95. Converges (Ratio Test)
96. Diverges (DT)
97. Diverges (Ratio Test)
98. Diverges (p-series + geom. series)
99. Diverges (Ratio Test)
100. Diverges (DT)
101. Converges (LCT)
102. Diverges (LCT)
103. Converges (CT)
104. Diverges (DT)
105. Converges (CT)
106. Diverges (DT)