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Calculus II – Final Exam Problems

**Limits**

Evaluate each limit.

1. $\lim_{x\to 0} \dfrac{x^2}{\ln\cos x}$
2. $\lim_{x\to 0} \dfrac{e^x - e^{-x} - 2x}{x - \sin x}$
3. $\lim_{x\to 0^+} (\tan x)\ln x$
4. $\lim_{x\to 0} \dfrac{\sin x - x\cos x}{x - \sin x}$
5. $\lim_{x\to 1^+} \left(\dfrac{1}{\ln x} - \dfrac{1}{x-1}\right)$
6. $\lim_{x\to 0} \dfrac{\tan x - \sin x}{x^3}$
7. $\lim_{x\to \frac{\pi}{2}^-} (2x\tan x - \pi\sec x)$
8. $\lim_{x\to 0^+} \left(\dfrac{1}{x-1} - \dfrac{2}{x^2-1}\right)$
9. $\lim_{x\to 0^+} \dfrac{e^{-1/x}}{x}$
10. $\lim_{x\to 0} \dfrac{\sin x - x\cos x}{x^2\sin x}$
11. $\lim_{x\to 0^+} \dfrac{x - \arctan x}{x^3}$
12. $\lim_{x\to\infty} \left(1 - \dfrac{1}{x}\right)^x$
13. $\lim_{x\to 0} \left(\dfrac{1}{x} - \dfrac{\ln|x+1|}{x^2}\right)$
14. $\lim_{x\to\infty} \left(1 - \sin\dfrac{1}{x}\right)^{2x}$
15. $\lim_{x\to\infty} x^2 e^{-x}$
16. $\lim_{x\to 0^+} \left(\dfrac{1}{\sin x} - \dfrac{1}{x}\right)$
17. $\lim_{x\to 2} \left(\dfrac{1}{\ln(x-1)} - \dfrac{1}{x-2}\right)$
18. $\lim_{x\to 0} \dfrac{\tan x - x}{x - \sin x}$
19. $\lim_{x\to 0^+} (1+2x)^{1/x^2}$
20. $\lim_{x\to 0^+} \dfrac{1 - \sin 2x}{x}$
21. $\lim_{x\to\infty} e^{-x}\ln x$
22. $\lim_{x\to 0^+} (1-2x)^{1/x}$
23. $\lim_{x\to 0^-} \dfrac{\cos(\frac{\pi}{2} - x)}{\arctan x}$
24. $\lim_{x\to 0^+} (1-2x)^{\cot x}$
25. $\lim_{x\to \frac{\pi}{2}^-} \dfrac{\sec x + 5}{3\tan x}$
26. $\lim_{x\to\infty} x\ln\left(1 + \dfrac{a}{x}\right)$
27. $\lim_{x\to\infty} x - \sqrt{x^2 - 10}$
28. $\lim_{x\to 0^+} (e^x + x)^{1/x}$
29. $\lim_{x\to\infty} x\sin\dfrac{1}{x}$
30. $\lim_{x\to \frac{\pi}{2}} (\sec x - \tan x)$
31. $\lim_{x\to 0^+} (x^x)^x$
32. $\lim_{x\to 0} \dfrac{e^x - e^{-x}}{\sin x}$
33. $\lim_{x\to 0^+} \left(\dfrac{1}{x} + \ln x\right)$
34. $\lim_{x\to\infty} \left(1 + \dfrac{1}{3x}\right)^x$
35. $\lim_{x\to 0} (1-2x)^{2/x}$
36. $\lim_{x\to 0^+} \dfrac{e^{2x}}{x\cos x}$
37. $\lim_{x\to 0} \dfrac{x - \sin x}{x - \tan x}$
38. $\lim_{x\to 0} (1 + \tan x)^{1/x}$
39. $\lim_{x\to 1} \left(\dfrac{1}{\ln x} - \dfrac{x}{\ln x}\right)$
40. $\lim_{x\to 0^+} \dfrac{\ln x}{\csc x}$
41. $\lim_{x\to 0^+} \left(\dfrac{2}{\sin^2 x} - \dfrac{1}{1-\cos x}\right)$
42. $\lim_{x\to\infty} \left(1 - \dfrac{3}{x}\right)^x$
43. $\lim_{x\to\infty} \dfrac{x^3}{e^x}$
44. $\lim_{x\to 0} (1 + \sin x)^{1/3x}$
45. $\lim_{x\to 0} \dfrac{xe^{2x}}{4\tan x}$
46. $\lim_{x\to 0^+} \left(\cot x - \dfrac{1}{x}\right)$
47. $\lim_{x\to 0^+} \left(\dfrac{1}{x}\right)^{\sin x}$
48. $\lim_{x\to\infty} \left(\dfrac{x+2}{x}\right)^x$
49. $\lim_{x\to 0^+} \left(\dfrac{1}{x} - \dfrac{1}{e^x - 1}\right)$
50. $\lim_{x\to 1^+} \left(\dfrac{1}{\ln x} + \dfrac{1}{x-1}\right)$
51. $\lim_{x\to 2\pi} \left(3 - \dfrac{x}{\pi}\right)^{\csc x}$
52. $\lim_{x\to -\frac{\pi}{2}} \left(\dfrac{\pi}{2} + x\right)\sec x$
53. $\lim_{x\to 1} \dfrac{1 - x + \ln x}{1 + \cos\pi x}$
54. $\lim_{x\to 0} (1+3x)^{\csc x}$

55. $\lim_{x\to\frac{\pi}{2}^-} (\sec x - \tan x)$

56. $\lim_{x\to\frac{\pi}{2}^-} (\cos x)\ln\cos x$

57. $\lim_{x\to 0} \dfrac{e^{2x} - e^{-2x} - 4x}{x^2}$

58. $\lim_{x\to\infty} (1+8x^2)^{1/x^2}$

59. $\lim_{x\to 0} \left[\dfrac{1}{\ln(1+x)} - \dfrac{1}{x}\right]$

60. $\lim_{x\to 0} \dfrac{1-\cos 2x}{x^2}$

61. $\lim_{x\to 0} (1+x)^{1/x}$

62. $\lim_{x\to\frac{\pi}{2}} \dfrac{\cot x - \cos x}{x^2}$

63. $\lim_{x\to 0^+} (1+2x)^{1/x}$

64. $\lim_{x\to 0^+} \dfrac{\ln\tan 2x}{\ln\sin 3x}$

65. $\lim_{x\to 0} (e^x + x)^{1/x}$

66. $\lim_{x\to\frac{\pi}{2}^-} \cos 3x \sec 7x$

67. $\lim_{x\to\infty} \left(1+\dfrac{3}{x}\right)^{2x}$

68. $\lim_{x\to 0} \dfrac{e^x - 1}{\sin x}$

69. $\lim_{x\to 0^+} \dfrac{\ln x}{\csc x}$

70. $\lim_{x\to 0^+} x^{1/\ln x}$

71. $\lim_{x\to 0^+} (\cot x - \ln x)$

72. $\lim_{x\to 0^+} (\sin x)^x$

73. $\lim_{x\to\frac{\pi}{2}^-} (\tan x - \sec x)$

74. $\lim_{x\to 0} \dfrac{x - \sin x}{x - \tan x}$

75. $\lim_{x\to\infty} \left(\dfrac{x}{x-1}\right)^{x/2}$

76. $\lim_{x\to\frac{\pi}{2}^-} \left[\dfrac{\sin x}{\cos x} + \dfrac{1}{x - \frac{\pi}{2}}\right]$

77. $\lim_{x\to-\infty} x^2 e^x$

78. $\lim_{x\to 0} \dfrac{\sin x - x\cos x}{x - \sin x}$

79. $\lim_{x\to 3^-} \left[\dfrac{1}{\ln(x-2)} - \dfrac{1}{x-3}\right]$

80. $\lim_{x\to 0^+} (1+3x^2)^{1/x^2}$

81. $\lim_{x\to 1} \dfrac{x - e^{x-1}}{(x-1)^2}$

82. $\lim_{x\to\infty} x\left[\dfrac{\pi}{2} - \arctan x\right]$

83. $\lim_{x\to\infty} \left(\dfrac{x}{x+2}\right)^{x+1}$

84. $\lim_{x\to 0^+} x^{\tan x}$

85. $\lim_{x\to 0^+} (\ln x)\tan x$

86. $\lim_{x\to 0} (1+\sin x)^{1/(2x)}$

87. $\lim_{x\to\pi} \dfrac{\sin x + x - \pi}{1+\cos x}$

88. $\lim_{x\to 0^+} \left(\dfrac{2}{x} - \dfrac{1}{1-\cos x}\right)$

89. $\lim_{x\to\frac{\pi}{2}^-} (2x\tan x - \pi\sec x)$

90. $\lim_{x\to\infty} (e^x + x)^{1/x}$

Answers:

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. | $-2$ | 2. | 2 | 3. | 0 | 4. | 2 | 5. | $\frac{1}{2}$ | 6. | $\frac{1}{2}$ | 7. | $-2$ | 8. | 1 |
| 9. | 0 | 10. | $\frac{1}{3}$ | 11. | $\frac{1}{3}$ | 12. | $\frac{1}{e}$ | 13. | $\frac{1}{2}$ | 14. | $\frac{1}{e^2}$ | 15. | 0 | 16. | 0 |
| 17. | $\frac{1}{2}$ | 18. | 2 | 19. | $\infty$ | 20. | $\infty$ | 21. | 0 | 22. | $\frac{1}{e^2}$ | 23. | 1 | 24. | $\frac{1}{e^2}$ |
| 25. | $\frac{1}{3}$ | 26. | $a$ | 27. | 0 | 28. | $e^2$ | 29. | 1 | 30. | 0 | 31. | 1 | 32. | 2 |
| 33. | $\infty$ | 34. | $e^{1/3}$ | 35. | $\frac{1}{e^4}$ | 36. | $\infty$ | 37. | $-\frac{1}{2}$ | 38. | $e$ | 39. | $-1$ | 40. | 0 |
| 41. | $\frac{1}{2}$ | 42. | $\frac{1}{e^3}$ | 43. | 0 | 44. | $e^{1/3}$ | 45. | $\frac{1}{4}$ | 46. | 0 | 47. | 1 | 48. | $e^2$ |
| 49. | $\frac{1}{2}$ | 50. | $\infty$ | 51. | $e^{-1/\pi}$ | 52. | 1 | 53. | $-\frac{1}{\pi^2}$ | 54. | $e^3$ | 55. | 0 | 56. | 0 |
| 57. | 0 | 58. | 1 | 59. | $\frac{1}{2}$ | 60. | 2 | 61. | $e$ | 62. | 0 | 63. | $e^2$ | 64. | 1 |
| 65. | $e^2$ | 66. | $\frac{3}{7}$ | 67. | $e^6$ | 68. | 1 | 69. | 0 | 70. | $e$ | 71. | $\infty$ | 72. | 1 |
| 73. | 0 | 74. | $-\frac{1}{2}$ | 75. | $e^{1/2}$ | 76. | 0 | 77. | 0 | 78. | 2 | 79. | $\frac{1}{2}$ | 80. | $e^3$ |
| 81. | $-\frac{1}{2}$ | 82. | 1 | 83. | $\frac{1}{e^2}$ | 84. | 1 | 85. | 0 | 86. | $e^{1/2}$ | 87. | 0 | 88. | $-\infty$ |
| 89. | $-2$ | 90. | $e$ | | | | | | | | | | | | |