# 1 Limits

Find the following limits, if they exist. If the limit does not exist, state the reason.

1. $\lim_{x\to 2} \dfrac{x^2-x-2}{x^2-3x+2}$
2. $\lim_{x\to 0} \dfrac{\pi}{4}$
3. $\lim_{x\to -\infty} \dfrac{-x^2+6x+8}{10x^2+10^6}$
4. $\lim_{x\to 3} \dfrac{\sqrt{x}-\sqrt{3}}{x-3}$
5. $\lim_{h\to 4} \dfrac{2x^2h-2xh+25h^2}{\sqrt{h}-5}$
6. $\lim_{h\to 0} \dfrac{\sqrt{2h+4}-2}{h}$
7. $\lim_{x\to 3^-} \dfrac{(2x+1)^3(x-5)^4}{(x-3)^3\sqrt{9-x}}$
8. $\lim_{x\to 0} \dfrac{x^2+2x}{x^3-3x}$
9. $\lim_{x\to \infty} \dfrac{2x^2+6x+3}{8+3x-5x^2}$
10. $\lim_{x\to 0} \dfrac{\sqrt{3-x}-\sqrt{3}}{x}$
11. $\lim_{x\to 2} \dfrac{x+3}{x-4}$
12. $\lim_{x\to 1} \dfrac{(x+2)(x-1)}{x^2-6x+5}$
13. $\lim_{x\to -\infty} \dfrac{3x^3+5x^2-7}{5x-4x^3}$
14. $\lim_{h\to 0} \dfrac{\sqrt{5+h}-\sqrt{5}}{h}$
15. $\lim_{x\to 3^-} \dfrac{x^2|x-3|}{x-3}$
16. $\lim_{x\to 2} \dfrac{x^2+3x-10}{x-2}$
17. $\lim_{x\to \infty} e^{-x}+1$
18. $\lim_{x\to \infty} \dfrac{2x^2-3x+7}{x^3-3}$
19. $\lim_{x\to 2} \dfrac{x^2-3x+2}{x^2+x-6}$
20. $\lim_{x\to -3^-} \dfrac{x-3}{x+3}$
21. $\lim_{x\to \infty} \dfrac{x-x^2}{x+x^2}$
22. $\lim_{x\to 0} \dfrac{x-x^2}{x+x^2}$
23. $\lim_{x\to -7} \dfrac{x^2+5x-14}{x+7}$
24. $\lim_{x\to \infty} \dfrac{12+2^x}{6-2^x}$
25. $\lim_{x\to \infty} \dfrac{2x-5}{x^2+x+1}$
26. $\lim_{x\to 4} \dfrac{x^2-x-12}{x-4}$
27. $\lim_{x\to -2} \dfrac{x^2+7x+10}{x+2}$
28. $\lim_{x\to \infty} \dfrac{6x^3+x^2+7x-2}{5x^2+2x-1}$
29. $\lim_{x\to \infty} \dfrac{e^x+e^{-x}}{e^x-e^{-x}}$
30. $\lim_{x\to 3^-} \dfrac{10x^3-x^2}{x-3}$
31. $\lim_{x\to -2} (2x^2-6x+5)^2$
32. $\lim_{x\to -5} \dfrac{x+5}{x^2+4x-5}$
33. $\lim_{x\to 0} \dfrac{(x+2)^2-4}{x}$
34. $\lim_{x\to 0^+} \pi$
35. $\lim_{h\to \infty} -2h$
36. $\lim_{y\to 2^-} \dfrac{(y-1)(y-2)}{y+1}$
37. $\lim_{x\to 1^+} \dfrac{x^4-1}{x-1}$
38. $\lim_{x\to -\infty} \dfrac{x-2}{x^2+2x+1}$
39. $\lim_{x\to 0} \dfrac{x}{|x|}$
40. $\lim_{h\to 0} \dfrac{\sin 2h}{h}$
41. $\lim_{t\to 2} \dfrac{t^2-t-2}{t^2-3t+2}$

42. $\lim_{h\to 3} \dfrac{h^2+h+1}{\sqrt{h+6}}$

43. $\lim_{x\to 0} \dfrac{\sqrt{2+x}-\sqrt{2-x}}{x}$

44. $\lim_{x\to\infty} \dfrac{3x^4-2x^2+1}{7x^4+6x^3+x}$

45. $\lim_{x\to-\infty} \dfrac{\sqrt{x^2+2x+2}}{x}$

46. $\lim_{x\to 2} \dfrac{x^2-2x+3}{\sqrt{8x}}$

47. $\lim_{h\to 0} \dfrac{\frac{1}{5+h}-\frac{1}{5}}{h}$

48. $\lim_{x\to 1^+} \dfrac{x^2+2x-3}{x^2-2x+1}$

49. $\lim_{x\to\infty} \dfrac{x^2-x^{-2}}{4x^2+4x^{-2}}$

50. $\lim_{r\to -1} \sqrt[3]{\dfrac{28+r}{r}}$

51. $\lim_{x\to-\infty} \dfrac{x^2+6x+5}{3x^2+4}$

52. $\lim_{x\to 9} \dfrac{\sqrt{x}-3}{x-9}$

53. $\lim_{x\to -2} \dfrac{(x+2)(x^2-3x+5)}{x^2+3x+2}$

54. $\lim_{x\to -3} \dfrac{x^2+x-6}{x^2-6x+8}$

55. $\lim_{x\to 0} \dfrac{\sqrt{x+9}-3}{x}$

56. $\lim_{x\to\infty} \dfrac{3x^3-4x^2+6x-5}{3+x-x^2-2x^3}$

57. $\lim_{x\to 0} \dfrac{x^3-1}{x-1}$

58. $\lim_{x\to 0} \dfrac{\sqrt{x+4}-2}{x}$

59. $\lim_{x\to 2} \dfrac{x^2+3x-10}{x^2-4}$

60. $\lim_{x\to 0} \dfrac{2-\sqrt{4-x}}{3x}$

61. $\lim_{x\to-\infty} \dfrac{\sqrt{x^2+1}}{3x-1}$

62. $\lim_{x\to\pi} \dfrac{2\cos^2 x-1}{\sin^2 x-1}$

63. $\lim_{x\to 3} \dfrac{(x-3)^2}{x^2-5x+6}$

64. $\lim_{t\to\infty} \dfrac{1-t^2+2t^3}{t-5t^3}$

65. $\lim_{x\to 0} \dfrac{\sin^2 3x}{x^2}$

66. $\lim_{x\to 7^+} \dfrac{3x-21}{x^2-x-42}$

67. $\lim_{x\to 0} \dfrac{3x}{\sin 5x}$

68. $\lim_{x\to-\infty} \dfrac{2-3x^2}{2x^2+5x-7}$

69. $\lim_{x\to 3^-} \dfrac{9-x^2}{x^2-6x+9}$

70. $\lim_{y\to\pi/2} \dfrac{\sin y}{\cos y-1}$

71. $\lim_{x\to 1} \dfrac{x^2+3x-4}{x-1}$

72. $\lim_{x\to 5^-} \dfrac{\sqrt{5-x}}{x-5}$

73. $\lim_{x\to 0} \dfrac{\sqrt{x+2}-\sqrt{2}}{x}$

74. $\lim_{x\to\infty} \dfrac{x^2+3x}{1-x^2-x^3}$

75. $\lim_{x\to 6} \dfrac{2x^2-12x}{x^2+x-42}$

76. $\lim_{x\to -4} \dfrac{3-\sqrt{25-x^2}}{x+4}$

77. $\lim_{x\to 0} \dfrac{x^2}{\sin^2 3x}$

78. $\lim_{x\to-\infty} \dfrac{x^3}{4+x^2}$

79. $\lim_{x\to 1} \dfrac{1-\sqrt{x}}{x-1}$

80. $\lim_{x\to 0} \dfrac{x}{\sin 4x}$

81. $\lim_{x\to\infty} \dfrac{2x^3+12x^2-18x}{x^3-x-12}$

82. $\lim_{x\to 5} \dfrac{x^2-3x-10}{x^2+2x-35}$

83. Given the function

$$f(x) = \begin{cases} \frac{8x^3+16x^2}{x+2} & \text{if } x < -\frac{1}{2} \\ 3 & \text{if } x = -\frac{1}{2} \\ \frac{\sin \pi x}{x} & \text{if } x > -\frac{1}{2} \end{cases}$$

Find the limits

(a) $\lim_{x\to 0} f(x)$

(b) $\lim_{x\to -\frac{1}{2}} f(x)$

(c) $\lim_{x\to -2^+} f(x)$

(d) $\lim_{x\to -\infty} f(x)$

84. Given the function

$$y = \frac{|x| - x}{x}$$

Find the limits

(a) $\lim_{x\to 0^+} y$

(b) $\lim_{x\to 0^-} y$

(c) $\lim_{x\to 0} y$

85. $\lim_{x\to -3} \sqrt{\dfrac{x^2-4x+4}{x^2-9x}}$

86. $\lim_{x\to 0} \dfrac{\sqrt{x+16}-4}{x}$

87. $\lim_{x\to 1} \dfrac{\ln x \sin(x-1)}{x-1}$

88. $\lim_{x\to \infty} \dfrac{x^3-7x^4}{3x^4-5x^3}$

89. $\lim_{y\to A} \dfrac{\frac{1}{A^2}-\frac{1}{y^2}}{y-A}$

90. $\lim_{x\to 0} \dfrac{\sin 3x}{x\cos x}$

91. $\lim_{x\to \infty} \dfrac{\sqrt{10x^2-5x+3}}{1-3x}$

92. $\lim_{x\to 2^-} \dfrac{(3x+4)^2}{x^2-x-2}$

93. $\lim_{x\to 2} \dfrac{4-x^2}{x^2-3x+2}$

94. Given the function

$$f(x) = \begin{cases} \frac{x^2-1}{x^2-x-2} & \text{if } x < 0 \\ 3 & \text{if } x = 0 \\ \frac{\sqrt{x}-2}{x-4} & \text{if } x > 0 \end{cases}$$

Evaluate the limits

(a) $\lim_{x\to 0} f(x)$

(b) $\lim_{x\to 4} f(x)$

(c) $\lim_{x\to -1} f(x)$

(d) $\lim_{x\to 2} f(x)$

(e) $\lim_{x\to -\infty} f(x)$

95. $\lim_{x\to 0} \dfrac{\sin 3x}{\sin 2x}$

96. $\lim_{x\to -\infty} \dfrac{4x+3}{3x^2+5}$

97. $\lim_{x\to -5^-} \dfrac{x^2-25}{x^2+10x+25}$

98. $\lim_{x\to 3} \dfrac{\sqrt{25-x^2}-4}{x-3}$

99. $\lim_{x\to -3} \dfrac{2x^2+5x-3}{3x^2+x-24}$

100. $\lim_{x\to 0} \left[\dfrac{x}{3\sin x} + \dfrac{\sin x}{3x}\right]$

101. $\lim_{x\to -\infty} \left[e^x + \dfrac{2x}{x+1}\right]$

102. $\lim_{x\to 4^+} \dfrac{2-x}{(x-4)^2}$

## Answers:

1. 3
2. $\frac{\pi}{4}$
3. $-\frac{1}{10}$
4. $\frac{1}{2\sqrt{3}}$
5. $\frac{8x^2-8x+400}{-3}$
6. $\frac{1}{2}$
7. $-\infty$
8. $-\frac{2}{3}$
9. $-\frac{2}{5}$
10. $-\frac{1}{2\sqrt{3}}$
11. $-\frac{5}{2}$
12. $-\frac{3}{4}$
13. $-\frac{3}{4}$
14. $\frac{1}{2\sqrt{5}}$
15. $-9$
16. 7
17. 1
18. 0
19. $\frac{1}{5}$
20. $\infty$
21. $-1$
22. 1
23. $-9$
24. $-1$
25. 0
26. 7
27. 3
28. $\infty$
29. 1
30. $-\infty$
31. 625
32. $-\frac{1}{6}$
33. 4
34. $\pi$
35. $-\infty$
36. 0
37. 4
38. 0
39. Does not exist
40. 2
41. 3
42. $\frac{13}{3}$
43. $\frac{1}{\sqrt{2}}$
44. $\frac{3}{7}$
45. $-1$
46. $\frac{3}{4}$
47. $-\frac{1}{25}$
48. $\infty$
49. $\frac{1}{4}$
50. $-3$
51. $\frac{1}{3}$
52. $\frac{1}{6}$
53. $-15$
54. 0
55. $\frac{1}{6}$
56. $-\frac{3}{2}$
57. 1
58. $\frac{1}{4}$
59. $\frac{7}{4}$
60. $\frac{1}{12}$
61. $-\frac{1}{3}$
62. $-1$
63. 0
64. $-\frac{2}{5}$
65. 9

66. $\frac{3}{13}$

67. $\frac{3}{5}$

68. $-\frac{3}{2}$

69. $\infty$

70. $-1$

71. 5

72. $-\infty$

73. $\frac{1}{2\sqrt{2}}$

74. 0

75. $\frac{12}{13}$

76. $-\frac{4}{3}$

77. $\frac{1}{9}$

78. $-\infty$

79. $-\frac{1}{2}$

80. $\frac{1}{4}$

81. 2

82. $\frac{7}{12}$

83. (a) $\pi$
    (b) 2
    (c) 32
    (d) $\infty$

84. (a) 0
    (b) $-2$
    (c) Does not exist

85. $\frac{5}{6}$

86. $\frac{1}{8}$

87. 0

88. $-\frac{7}{3}$

89. $\frac{2}{A^3}$

90. 3

91. $-\frac{\sqrt{10}}{3}$

92. $-\infty$

93. $-4$

94. (a) $\frac{1}{2}$
    (b) $\frac{1}{4}$
    (c) $\frac{2}{3}$
    (d) $\frac{2-\sqrt{2}}{2}$
    (e) 1

95. $\frac{3}{2}$

96. 0

97. $\infty$

98. $-\frac{3}{4}$

99. $\frac{7}{17}$

100. $\frac{2}{3}$

101. 2

102. $-\infty$