1 Differential Equations

1. Find $f(x)$ given $f''(x) = 3\sqrt{x}$, $f'(1) = -1$, and $f(0) = 32$.
2. Find $f(x)$ fiven $f''(x) = 2x^{-1/3}$, $f'(1) = 4$, and $f(0) = 2$.
3. Solve the equation $y' = e^x$.
4. Solve the equation $\dfrac{d^2y}{dx^2} = x$ given that $y = 7$ when $x = 0$ and that $\dfrac{dy}{dx} = 4$ when $x = 2$.
5. If $\dfrac{d^2y}{dx^2} = -2x + 1$ find y in terms of x, given that $\dfrac{dy}{dx} = 1$ and $y = -1$ when $x = 0$.
6. If $y'' = 1 - 6x$ find y in terms of x given that $y' = -5$ and $y = \frac{5}{2}$ when $x = -1$.
7. Find the function $s(t)$ which satisfies the conditions: $s''(t) = e^t$; $s'(0) = 0$; $s(0) = 0$.
8. Find $f(x)$ in terms of x given that $f''(x) = x^2 + 3x$, and that $f'(1) = 1$ and $f(1) = 2$.
9. Find $f(x)$ given that $f''(x) = \dfrac{6}{x^2}$ and $f'(\frac{1}{3}) = 2$ and $f(1) = 5$.
10. If $f''(x) = -6x$; $f'(2) = -14$; $f(2) = -9$ find $f(x)$.
11. A point $P(x, y)$ moves in a plane such that $\dfrac{dx}{dt} = \dfrac{1}{t}$ and $\dfrac{dy}{dt} = 2t - 4$ fot $t \geq 0$.
    (a) Express x and y as functions of t if $x = \ln 2$ and $y = 1$ when $t = 2$.
    (b) Express y as a function of x.
12. Given $f'(x) = 5x + \dfrac{6}{x^2}$ and $f(2) = 3$; find $f(x)$.

Answers:

1. $\frac{4}{5}x^{5/2} - 3x + 32$
2. $\frac{9}{5}x^{5/3} + x + 2$
3. $e^x + C$
4. $\frac{1}{6}x^3 + 2x + 7$
5. $-\frac{1}{3}x^3 + \frac{1}{2}x^2 + x - 1$
6. $\frac{1}{2}x^2 - x^3 - x$
7. $e^t - t - 1$
8. $\frac{1}{12}x^4 + \frac{1}{2}x^3 - \frac{5}{6}x + \frac{9}{4}$
9. $-6\ln x + 20x - 15$
10. $-x^3 - 2x + 3$
11. (a) $x = \ln t$ and $y = t^2 - 4t + 5$
    (b) $y = e^{2x} - 4e^x + 5$
12. $\frac{5}{2}x^2 - \dfrac{6}{x} - 4$