1 Curve Sketching

For the each problem determine the following if applicable:

(a) x- and y-intercepts

(b) vertical and horizontal asymptotes

(c) open intervals on which the function is increasing and decreasing

(d) open intervals where the function is concave up and concave down

(e) coordinates of all relative extrema

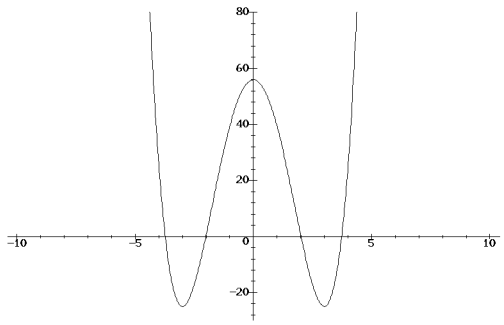
(f) coordinates of all points of inflection

(g) sketch the curve, showing required information

1. $y = x^4 - 18x^2 + 56$
2. $y = \dfrac{2x - x^2}{x^2 - 2x + 1}$
3. $y = x^5 - 5x$
4. $y = \dfrac{(x-3)(x+2)}{(x+1)(x-2)}$
5. $y = x^4 - 4x^3$
6. $y = x^3 - 9x^2 + 15x - 5$
7. $y = x^3 - 3x^2 + 3$
8. $y = 2x^3 - 12x^2 + 18x$
9. $y = 2x^4 - x^2$
10. $y = \frac{1}{4}(x^3 - \frac{3}{2}x^2 - 6x + 2)$
11. $y = x^5 - 5x^4$
12. $y = 8 - 9x + 6x^2 - x^3$
13. $y = \dfrac{9x}{(3x+1)^2}$
    $y' = \dfrac{9(1-3x)}{(3x+1)^3} \qquad y'' = \dfrac{54(3x-2)}{(3x+1)^4}$
14. $y = \dfrac{x^2}{x^2-1}$
    $y' = \dfrac{-2x}{(x^2-1)^2} \qquad y'' = \dfrac{2(3x^2+1)}{(x^2-1)^3}$
15. $y = \dfrac{x}{(x+1)^2}$
    $y' = \dfrac{1-x}{(x+1)^3} \qquad y'' = \dfrac{2(x-2)}{(x+1)^4}$
16. $y = \dfrac{x^2-1}{x^3}$
    $y' = \dfrac{-x^2+3}{x^4} \qquad y'' = \dfrac{2x^2-12}{x^5}$
17. $y = \dfrac{x+1}{x^2}$
    $y' = \dfrac{-x-2}{x^3} \qquad y'' = \dfrac{2x+6}{x^4}$
18. $f(x) = x^{2/3}(x-5)$
    $f'(x) = \dfrac{5(x-2)}{3x^{1/3}} \qquad f''(x) = \dfrac{10(x+1)}{9x^{4/3}}$
19. $y = \dfrac{1}{x^2-x}$
    $y' = \dfrac{1-2x}{(x^2-x)^2} \qquad y'' = \dfrac{2(3x^2-3x+1)}{(x^2-x)^3}$
20. $y = x(16-x^2)^{1/2}$
    $y' = 2(16-x^2)^{-1/2}(8-x^2)$
    $y'' = -2x(16-x^2)^{-3/2}(24-x^2)$
21. $f(x) = 18(x^{-2} + x^{-1}) = \dfrac{18(x+1)}{x^2}$
22. $f(x) = x^{1/3}(x+1)$
    $f'(x) = \dfrac{4x+1}{2x^{2/3}} \qquad f''(x) = \dfrac{2(2x-1)}{9x^{5/3}}$
23. $f(x) = \dfrac{-6x}{x^2+1}$
    $f'(x) = \dfrac{6(x+1)(x-1)}{(x^2+1)^2}$
    $f''(x) = \dfrac{-12x(x^2-3)}{(x^2+1)^3}$
24. $y = 5x^{2/3} - x^{5/3}$
25. $f(x) = \dfrac{6-6x}{x^2}$
    $f'(x) = \dfrac{6x-12}{x^3} \qquad f''(x) = \dfrac{36-12x}{x^4}$
26. $f(x) = 1 + \dfrac{1}{x} + \dfrac{1}{x^2}$
    $f'(x) = -\dfrac{1}{x^2} - \dfrac{2}{x^3} \qquad f''(x) = \dfrac{2}{x^3} + \dfrac{6}{x^4}$
27. $y = \dfrac{x^2-1}{x^3}$
    $y' = \dfrac{3-x^2}{x^4} \qquad y'' = \dfrac{2(x^2-6)}{x^5}$
28. $y = x^4 - 6x^2 + 8x + 10$
    $\dfrac{dy}{dx} = 4(x-1)^2(x+2)$

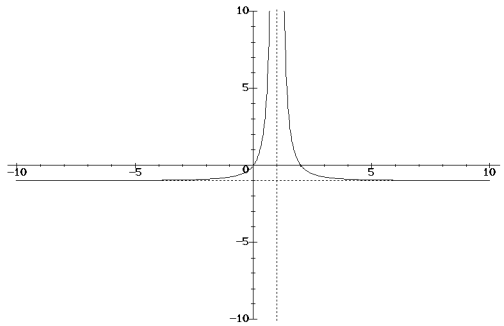
Answers: 1–4.

1.



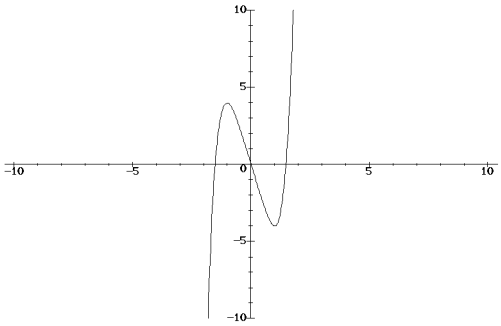
x-intercepts	$(\pm 2, 0); (\pm\sqrt{14}, 0)$
y-intercept	$(0, 56)$
H.A.	none
V.A.	none
Increasing on	$(-3, 0), (3, \infty)$
Decreasing on	$(-\infty, -3), (0, 3)$
Concave up on	$(-\infty, -\sqrt{3}), (\sqrt{3}, \infty)$
Concave down on	$(-\sqrt{3}, \sqrt{3})$
Rel. Max. at	$(0, 56)$
Rel. Min. at	$(-3, -25); (3, -25)$
Inflection pt. at	$(-\sqrt{3}, 11); (\sqrt{3}, 11)$

2.



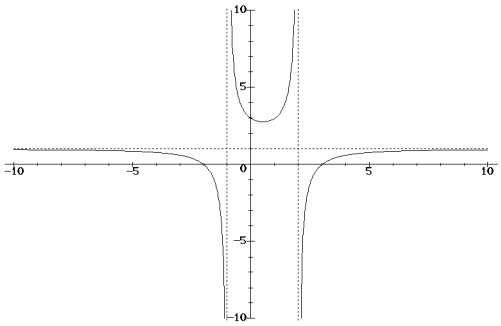
x-intercepts	$(0, 0); (2, 0)$
y-intercept	$(0, 0)$
H.A.	$y = -1$
V.A.	$x = 1$
Increasing on	$(-\infty, 1)$
Decreasing on	$(1, \infty)$
Concave up on	$(-\infty, 1), (1, \infty)$
Concave down on	none
Rel. Max. at	none
Rel. Min. at	none
Inflection pt. at	none

3.



x-intercepts	$(0, 0); (-\sqrt[4]{5}, 0); (\sqrt[4]{5}, 0)$
y-intercept	$(0, 0)$
H.A.	none
V.A.	none
Increasing on	$(-\infty, -1), (1, \infty)$
Decreasing on	$(-1, 1)$
Concave up on	$(0, \infty)$
Concave down on	$(-\infty, 0)$
Rel. Max. at	$(-1, 4)$
Rel. Min. at	$(1, -4)$
Inflection pt. at	$(0, 0)$

4.



x-intercepts	$(-2, 0); (3, 0)$
y-intercept	$(0, 3)$
H.A.	$y = 1$
V.A.	$x = -1; x = 2$
Increasing on	$(\frac{1}{2}, 2), (2, \infty)$
Decreasing on	$(-\infty, -1), (-1, \frac{1}{2})$
Concave up on	$(-1, 2)$
Concave down on	$(-\infty, -1), (2, \infty)$
Rel. Max. at	none
Rel. Min. at	$(\frac{1}{2}, \frac{25}{9})$
Inflection pt. at	none