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(May 2016)

Test 3
(version A)

Calculus III (Maths 201–DDB)

Justify all your answers — just having the correct answer is not sufficient.
Pace yourself — a rough guide is to spend not more than $2m$ minutes or so on a question worth m marks.

(Marks)

(3) 1. (a) If $z\,e^{xy} + 2xz = 4x^2y + 3$ defines $z = f(x, y)$, find $\dfrac{\partial z}{\partial y}$.

(3) (b) Suppose $z = f(y^2 - x^2, x^2 - y^2)$, f a differentiable function, show that $y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = 0$.

2. Suppose $f(x, y) = 3xy - x^3 + y^3$, and P_0 is the point $(1, -1)$.

(2) (a) Find the equation of the tangent plane to the surface $z = f(x, y)$ at P_0.

(2) (b) What is the direction of greatest increase in f at P_0?

(2) (c) At what rate is $f(x, y)$ changing if (x, y) moves in a straight line *from* P_0 *to* the origin?

(4) (d) Find and classify the critical points of $f(x, y)$.

(4) (e) Find the absolute maximum and the absolute minimum values of $f(x, y)$ in the closed finite region bounded by the parabola $y = x^2$ and the line $y = 4$.

(6) 3. Use Lagrange Multipliers to find the maximum and minimum values of the sum $x^2 + y^2 + z^2$ for a point (x, y, z) which lies on the plane $x + 2y + 3z = 7$.

(2×6) 4. Evaluate the following: (change coordinates as appropriate).

(a) $\displaystyle\int_0^2 \int_{x^2}^4 \sqrt{y}\,\cos(y^2)\,dy\,dx$

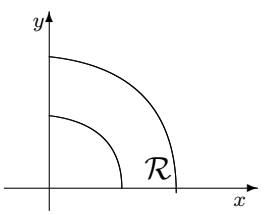
(b) $\displaystyle\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{-\sqrt{1-x^2-y^2}}^{\sqrt{1-x^2-y^2}} \frac{1}{\sqrt{x^2+y^2}}\,dz\,dy\,dx$ *(Hint: spherical)*

(6) 5. Evaluate the following sum of double integrals

$$\int_0^2 \int_0^x \sqrt{x^2+y^2}\,dy\,dx + \int_2^{2\sqrt{2}} \int_0^{\sqrt{8-x^2}} \sqrt{x^2+y^2}\,dy\,dx$$

by combining the sum into a single integral in polar coordinates.

(6) 6. Use the transformation $\{\, x = e^u \cos v\,,\ y = e^u \sin v\,\}$ to convert the integral $\displaystyle\iint_{\mathcal{R}} dA$ to a uv integral, where $\mathcal{R}$ is the region bounded by the lines $x = 0$, $y = 0$, $x^2 + y^2 = 1$, $x^2 + y^2 = e^2$. Evaluate the uv integral.



(Total: 50)