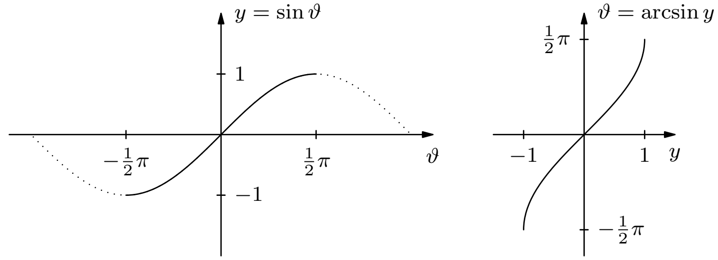
The inverse sine function. The sine function restricted to $[\,-\tfrac{1}{2}\pi,\,\tfrac{1}{2}\pi\,]$ is one-to-one, and its inverse on this interval is called the *arcsine* $(\arcsin)$ *function*. The domain of $\arcsin$ is $[\,-1,1\,]$ and the range of $\arcsin$ is $[\,-\tfrac{1}{2}\pi,\,\tfrac{1}{2}\pi\,]$. Below is a graph of $y=\sin\vartheta$, with the the part over $[\,-\tfrac{1}{2}\pi,\,\tfrac{1}{2}\pi\,]$ emphasized, and the graph of $\vartheta=\arcsin y$.



By definition,

$$\vartheta=\arcsin y \quad\text{means that}\quad \sin\vartheta=y \quad\text{and}\quad -\tfrac{1}{2}\pi\leqslant\vartheta\leqslant\tfrac{1}{2}\pi.$$

Differentiating the equation on the right implicitly with respect to y, gives

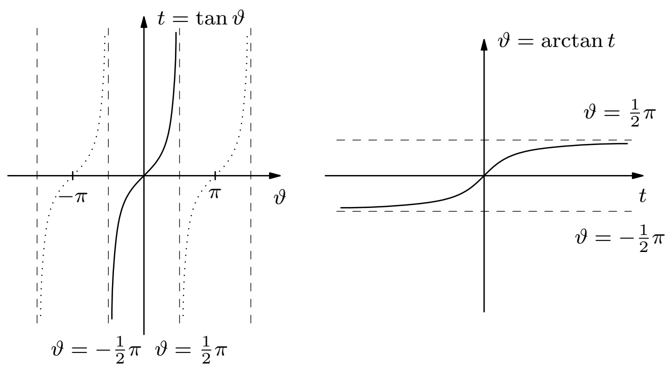
$$\cos\vartheta\frac{d\vartheta}{dy}=1,\quad\text{or}\quad \frac{d\vartheta}{dy}=\frac{1}{\cos\vartheta},\quad\text{provided}\quad -\tfrac{1}{2}\pi<\vartheta<\tfrac{1}{2}\pi.$$

Since $\cos\vartheta>0$ on $(\,-\tfrac{1}{2}\pi,\,\tfrac{1}{2}\pi\,)$, it follows that $\cos\vartheta=\sqrt{1-\sin^2\vartheta}=\sqrt{1-y^2}$. Therefore,

$$\frac{d}{dy}(\arcsin y)=\frac{1}{\sqrt{1-y^2}},\quad\text{for}\quad -1<y<1.$$

Observe that the graph of $\vartheta=\arcsin y$ has vertical tangents where $y=\pm1$.

The inverse tangent function. The tangent function restricted to $(\,-\tfrac{1}{2}\pi,\,\tfrac{1}{2}\pi\,)$ is one-to-one, and its inverse on this interval is called the *arctangent* $(\arctan)$ *function*. The domain of $\arctan$ is $\mathbb{R}$ and the range of $\arctan$ is $(\,-\tfrac{1}{2}\pi,\,\tfrac{1}{2}\pi\,)$. Below is a graph of $y=\tan\vartheta$, with the the part over $(\,-\tfrac{1}{2}\pi,\,\tfrac{1}{2}\pi\,)$ emphasized, and the graph of $\vartheta=\arctan y$.



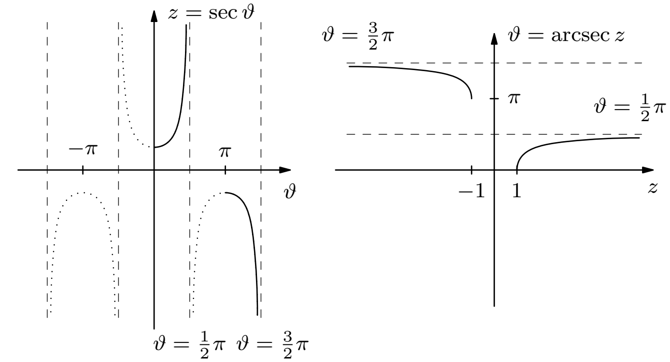
By definition,

$$\vartheta=\arctan t \quad\text{means that}\quad \tan\vartheta=t \quad\text{and}\quad -\tfrac{1}{2}\pi<\vartheta<\tfrac{1}{2}\pi.$$

As with $\arcsin$, one can use the equation on the right, *and the restriction on* ϑ, to compute the derivative of the arctangent function:

$$\frac{d}{dt}(\arctan t)=\frac{1}{1+t^2},\quad\text{for}\quad t\in\mathbb{R}.$$

The inverse secant function. The secant function restricted to $[\,0,\,\tfrac{1}{2}\pi\,)\cup[\,\pi,\,\tfrac{3}{2}\pi\,)$ is one-to-one, and its inverse on this interval is called the *arcsecant* (arcsec) *function*. The domain of arcsec is $(\,-\infty,-1\,]\cup[\,1,\infty\,)$ and the range of arcsec is $[\,0,\,\tfrac{1}{2}\pi\,)\cup[\,\pi,\,\tfrac{3}{2}\pi\,)$. Below is a graph of $z=\sec\vartheta$, with the the part over $[\,0,\,\tfrac{1}{2}\pi\,)\cup[\,\pi,\,\tfrac{3}{2}\pi\,)$. emphasized, and the graph of $\vartheta=\operatorname{arcsec} z$.



By definition,

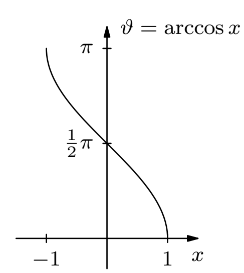
$$\vartheta=\operatorname{arcsec} z \quad\text{means that}\quad \sec\vartheta=t \quad\text{and}\quad 0\leqslant\vartheta<\tfrac{1}{2}\pi \quad\text{or}\quad \pi\leqslant\vartheta<\tfrac{3}{2}\pi.$$

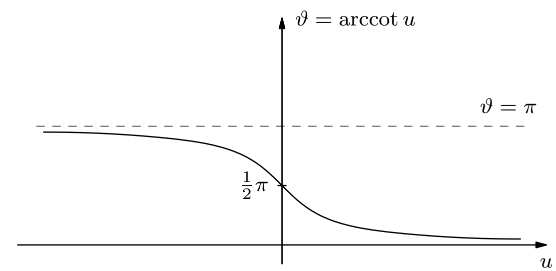
Again, differentiating the equation on the right implicitly with respect to z, *and using the restriction in* ϑ, one computes the derivative

$$\frac{d}{dz}(\operatorname{arcsec} z)=\frac{1}{z\sqrt{z^2-1}},\quad\text{for}\quad |z|>1.$$

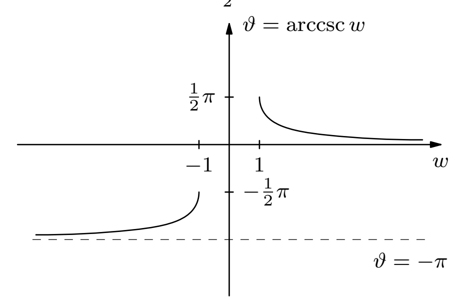
The other inverse trigonometric functions. The remaining trigonometric functions can be defined via the cofunction identities. Below are their definitions and their graphs. *Note* that the domain restriction on the cosecant function given here differs with that of the textbook (in fact there are three common choices). In practice this will not matter, but you should use this definition in your written work and for WeBWorK.

$$\arccos x=\tfrac{1}{2}\pi-\arcsin x \qquad\qquad \operatorname{arccot} u=\tfrac{1}{2}\pi-\arctan u$$





$$\operatorname{arccsc} w=\tfrac{1}{2}\pi-\operatorname{arcsec} w$$



What is the domain and range of each of these functions? You should try to derive the formulæ for the derivatives of the inverse trigonometric functions besides $\arcsin$. (Note that computing the derivatives of the last three will be easier.) Finally, identify any vertical tangents, and/or horizontal asymptotes of the graphs of each of the inverse trigonometric functions.