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Test 3
(Version B)

Cal I (S) (Maths 201–NYA)

(Marks)

Justify *all your answers—just having the correct answer is not sufficient.*
Pace yourself—a rough guide is to spend not more than $2m$ minutes or so on a question worth m marks.

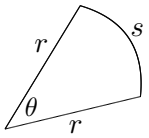
(4) 1. For $y = x^3\sqrt{2x+1}$ calculate y' and y'' and simplify them both *completely*, so that your answers are simple fractions.

(7) 2. For the function $y = (x+2)(x-2)^{1/3}$, what is the domain of the function? Identify all intercepts, aymptotes, local extrema, and inflection points. Specify intervals (*e.g.* in a table of data) where the graph is increasing, decreasing, concave up, and concave down. Show all your work. To help you on your way, the first and second derivatives are already calculated for you. You may also find it useful to know that $\sqrt[3]{2}$ is approximately $\frac{5}{4}$.
Now: sketch the function.

$$y = (x+2)(x-2)^{1/3} \qquad y' = \frac{4(x-1)}{3(x-2)^{2/3}} \qquad y'' = \frac{4(x-4)}{9(x-2)^{5/3}}$$

(2) 3. Consider the function $f(x) = x^{-1}$ on the interval $[-1, 1]$: show that although $f(-1) = -1$, $f(1) = 1$, and so $\frac{f(1)-f(-1)}{1-(-1)} = 1$, nonetheless there is no value $x = c$ between -1 and 1 so that $f'(c) = 1$. Why does this not contradict the Mean Value Theorem. *(State the Mean Value Theorem clearly, and show why this function does not contradict it.)*

(5) 4. A slice of pizza, in the form of a sector of a circle, is to have perimeter of 60 cm. What should the radius of the pan be in order to make a slice of largest area? (Some formulas: the area of a sector of a circle is $A = \frac{1}{2}\theta r^2$ and the arc length of a sector is $s = r\theta$.)



(3) 5. Find the function $f(x)$ satisfying $f''(x) = 6x + \dfrac{4}{x^2} + 1$, $f(1) = 0$, $f'(1) = 3$.

(3) 6. Approximate the value of the integral $\displaystyle\int_0^2 x\, 2^{2x}\, dx$ using a Riemann sum with 4 equal subintervals, using right endpoints (the "end" of each subinterval).

(5×4) 7. Evaluate the following:

(a) $\displaystyle\int \frac{3x^2 - 2x + 5}{\sqrt[4]{x}}\, dx$

(b) $\displaystyle\int (\pi^2 + \sqrt[6]{x} - \mathrm{e}^x)\, dx$

(c) $\displaystyle\int_{\pi/4}^{\pi/2} \frac{1 - \cos x}{\sin^2 x}\, dx$

(d) $\displaystyle\int_{-1}^{1} |t^2 - 1|\, dt$

(e) $\displaystyle\int_{-1}^{1} x^3 \cos(x)\, dx$ *(Hint: symmetry?)*

(3) 8. Given the function $f(x) = \displaystyle\int_1^{x^3} \frac{t}{\tan(t) + \mathrm{e}^t}\, dt$ what is $f'(x)$?

(3) 9. Find the area of one arch of the cosine function: *i.e.* find the area between $y = 0$, $y = \cos x$, $x = -\pi$, and $x = \pi$.

(Total: 50)