Instructor: Dr. R.A.G. Seely (May 2010)

Test 3 (version A)

Calculus III (Maths 201–DDB)

Justify all your answers — just having the correct answer is not sufficient.
Pace yourself — a rough guide is to spend not more than 2m minutes or so on a question worth m marks.

(6) 1. Suppose $F(x, y, z) = x^2 y - 2x + y - z^2$.

(a) Find the directional derivative of F at the point $P_0(1, 2, -11)$ in the direction **from** P_0 **to** the origin.

(b) For the level surface (contour surface) $F(x, y, z) + 119 = 0$ find the equation of the tangent plane at P_0.

(c) Determine the maximum rate of increase in F at P_0; in what direction does the maximum rate occur?

(4) 2. Suppose $z = f(x^2 - y^2, y^2 - x^2)$, f a differentiable function, show that $y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = 0$.

(6) 3. Find and classify the critical points of $f(x, y) = 4xy - 2x^4 - y^2$.

(6) 4. Use Lagrange Multipliers to find the extreme (max and min) values of $f(x, y, z) = 3x - y - 3z$ subject to the constraints $z = x + y$ and $x^2 + 2z^2 = 18$.

(3×6) 5. Evaluate the following: (change coordinates as appropriate).

(a) $\displaystyle\int_0^1 \int_x^1 e^{-y^2}\, dy\, dx$

(b) $\displaystyle\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{1}{1 + x^2 + y^2}\, dy\, dx$

(c) $\displaystyle\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \sqrt{x^2 + y^2 + z^2}\, dz\, dy\, dx$

(6) 6. Sketch the solid region $\mathcal{S}$ that lies inside both the cylinder $x^2 + y^2 = 4$ and the ellipsoid $2x^2 + 2y^2 + z^2 = 18$. Find the volume of $\mathcal{S}$.

(4) 7. Use the transformation $\{\, u = x + y\,,\ v = x - y \,\}$ to evaluate the integral $\displaystyle\iint_{\mathcal{R}} e^{x+y}\, dA$, where $\mathcal{R}$ is the region bounded by the lines $x + y = 0$, $x + y = 1$, $x - y = 0$, $x - y = 1$.

(Total: 50)