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Test 3

(version A)

## Calculus III (Maths 201–DDB)

*Justify all your answers — just having the correct answer is not sufficient.*
*Pace yourself — a rough guide is to spend not more than $2m$ minutes or so on a question worth $m$ marks.*

(6) 1. Suppose $F(x, y, z) = x^2y - 2x + y - z^2$.

(a) Find the directional derivative of $F$ at the point $P_0(1, 2, -11)$ in the direction **from** $P_0$ **to** the origin.

(b) For the level surface (contour surface) $F(x, y, z) + 119 = 0$ find the equation of the tangent plane at $P_0$.

(c) Determine the maximum rate of increase in $F$ at $P_0$; in what direction does the maximum rate occur?

(4) 2. Suppose $z = f(x^2 - y^2, y^2 - x^2)$, $f$ a differentiable function, show that $y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = 0$.

(6) 3. Find and classify the critical points of $f(x, y) = 4xy - 2x^4 - y^2$.

(6) 4. Use Lagrange Multipliers to find the extreme (max and min) values of $f(x, y, z) = 3x - y - 3z$ subject to the constraints $z = x + y$ and $x^2 + 2z^2 = 18$.

(3×6) 5. Evaluate the following: (change coordinates as appropriate).

(a) $\displaystyle\int_0^1 \int_x^1 \mathrm{e}^{-y^2}\, dy\, dx$

(b) $\displaystyle\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{1}{1 + x^2 + y^2}\, dy\, dx$

(c) $\displaystyle\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \sqrt{x^2 + y^2 + z^2}\, dz\, dy\, dx$

(6) 6. Sketch the solid region $\mathcal{S}$ that lies inside both the cylinder $x^2 + y^2 = 4$ and the ellipsoid $2x^2 + 2y^2 + z^2 = 18$. Find the volume of $\mathcal{S}$.

(4) 7. Use the transformation $\{\, u = x + y\,,\ v = x - y \,\}$ to evaluate the integral $\displaystyle\iint_{\mathcal{R}} \mathrm{e}^{x+y}\, dA$, where $\mathcal{R}$ is the region bounded by the lines $x + y = 0$, $x + y = 1$, $x - y = 0$, $x - y = 1$.

(Total: 50)