Instructor: Dr. R.A.G. Seely
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Test 3
(version A)

Calculus III (Maths 201–DDB)

(Marks)

(8) 1. Suppose $F(x, y, z) = xz^2 - yz^3 + \cos(xy)$.

(a) Find the gradient of F at the point $P_0(0, 8, -2)$.

(b) For the level surface (contour surface) $F(x, y, z) = 65$, find the equation of the tangent plane at P_0.

(c) On the level surface $F(x, y, z) = 65$ find $\dfrac{\partial z}{\partial y}$.

(d) If $z = f(x, y)$ is implicitly determined by the level surface $F(x, y, z) = 65$ and $f(0, 8) = -2$, calculate $\nabla f\,(0, 8)$, and use it (or the answer to (1b) above) to give an estimate of $f(0.1, 7.9)$.

(6) 2. Suppose $f(t)$ is differentiable, and let $z = yf(x^2 - y^2)$; show that

$$y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = \frac{xz}{y}$$

(6) 3. Find and classify the critical points of $f(x, y) = x^3 + 3xy^2 + 3y^2 - 15x + 2$.

(6) 4. Use Lagrange Multipliers to find the extreme values of $f(x, y, z) = x^2 + y^2 + z^2$ where (x, y, z) lie on the plane $x - 2y + 3z = 7$.

(6) 5. Evaluate the following (change coordinates as appropriate):

$$\int_0^1 \int_{\sqrt{y}}^1 \frac{1}{2 - x^3}\, dx\, dy$$

(6) 6. Sketch the solid region of integration for the following:

$$\int_0^{2\pi} \int_0^{\sqrt{3}} \int_{r/\sqrt{3}}^{\sqrt{4-r^2}} r\sqrt{r^2 + z^2}\, dz\, dr\, d\theta$$

Convert the integral to spherical coordinates. *(Hint: Remember the 1-$\sqrt{3}$-2 triangle.)*
Evaluate the triple integral by whatever method you prefer.

(6) 7. Sketch the solid region $\mathcal{S}$ that lies inside both the cylinder $x^2 + y^2 = 4$ and the ellipsoid $2x^2 + 2y^2 + z^2 = 18$. Find the volume of $\mathcal{S}$.

(6) 8. Use the transformation $\{\, x = u + v\,,\ y = u - 3v \,\}$ to evaluate the integral $\iint_{\mathcal{R}} \sqrt{3x + y}\, dA$, where $\mathcal{R}$ is the region bounded by the lines $3x + y = 0$, $3x + y = 4$, $x - y = 0$, $x - y = 8$.

(Total: 50)

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