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Test 2
(version A)

Calculus III (Maths 201–DDB)

Justify all your answers — just having the correct answer is not sufficient.
Pace yourself — a rough guide is to spend not more than 2m minutes or so on a question worth m marks.

(3) 1. (a) When and where was Kepler born? What university did he attend, and roughly when did he attend it?
(b) In his early description of his cosmology, Kepler used several geometric solids (3D shapes) in a manner that seems very odd to modern readers; what were these solids, and what role did they play in his theory?

(2×2) 2. (a) Find the equation of the plane through $(1, 1, 1)$ that intersects the xy-plane in the same line as the plane $3x + 2y - z = 6$.
(b) Find the angle between the vector joining (a, b, c) and (b, c, a) and the vector joining (a, b, c) and (c, a, b). (Note: show this angle is the same, no matter what a, b, c are.)
(Hint: calculate the appropriate dot product carefully, and all will work out just fine. There is also a geometric solution, but don't spend too long on this question.)

(9) 3. Name and sketch the following surfaces in 3-space. Show all your work, including traces, intercepts (and contour curves if you use them).
(a) $z + 4x^2 = y^2$ (b) $z^2 = r^2 + 1$ (c) $\rho = 4 \csc \phi$

(2) 4. Find the parametrization of $\boldsymbol{r}(t) = \langle 4 \cos t\,,\, 3\,,\, 4 \sin t\rangle$ in terms of the arclength s.
Use $t = 0$ for the "starting point".

(4) 5. For the conical helix $\boldsymbol{r} = \langle t \cos t, t, t \sin t\rangle$:
(a) What is the equation of the cone on which this curve lies? *(Hint: Pythagoras)*
(b) Sketch the graphs of the cone and the part of the helix from $t = 0$ to $t = 2\pi$, indicating the direction of increasing t.

(3) 6. An object moves along a curve $\boldsymbol{r} = \boldsymbol{r}(t)$ so that for all t, the velocity and acceleration $\boldsymbol{v}$, $\boldsymbol{a}$ are always unit vectors. What is the curvature κ of the path of the curve?

(11) 7. A particle P moves along a curve $\boldsymbol{r}(t) = t\,\boldsymbol{i} + t^2\,\boldsymbol{j} + \frac{2}{3}t^3\,\boldsymbol{k}$. Find the unit tangent vector $\boldsymbol{T}(t)$, the unit normal vector $\boldsymbol{N}(t)$, the curvature $\kappa(t)$, the tangential and the normal components a_T, a_N of acceleration.

(Hint: do the algebra carefully, and you will find there is a lot of simplification—all the square roots work out easily if you factor where appropriate.)

(6) 8. Find the (radius, center, and) equation of the osculating circle for the curve $y = \sec x$ at the point $(0, 1)$. Draw a graph of the curve and the osculating circle (on the same axes) near $(0, 1)$.

(4) 9. Calculate the following limits (if possible; if the limit does not exist, say so, and justify your answer).
(a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^4y^2}{x^2 + y^4}$ (b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^4y^2}{x^8 + y^4}$

(4) 10. Show that $z = \sin x \cos ay$ is a solution of the equation $\dfrac{\partial^2 z}{\partial x^2} = \dfrac{1}{a^2}\dfrac{\partial^2 z}{\partial y^2}$.

(Total: 50)

([3]) **Bonus:** For a space curve given by a vector function $\boldsymbol{r} = \boldsymbol{r}(t)$, show that if $\boldsymbol{a}$ is parallel to $\boldsymbol{r}$, then the path of the space curve is planar, (where as usual $\boldsymbol{a}$ is the acceleration vector).