Calculus III (Maths 201–DDB)

Answers

1. $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{x^{4n+1}}{(4n+1)(2n+3)!} \simeq \frac{1}{12}-\frac{1}{19200}\pm\frac{1}{23224320} = 0.08328125 \pm 4\times10^{-8}$

2. (a) $1+\displaystyle\sum_{n=1}^{\infty}\frac{(2n-1)!!}{2^n\,n!}x^{2n}$, IofC: $(-1,1)$ (b) $x+\displaystyle\sum_{n=1}^{\infty}\frac{(2n-1)!!\,x^{2n+1}}{2^n\,n!\,(2n+1)}$, RofC: 1
   (c) $\pi/6 = \arcsin(\frac{1}{2}) \simeq \frac{1}{2}+\frac{1}{48}+\frac{3}{1280}-\frac{15}{2^{10}\cdot6\cdot7} = \frac{112579}{215040} = 0.524$

3. $T_3(x) = 2[1+\frac{1}{160}(x-32)-\frac{1}{12800}(x-32)^2+\frac{3}{2048000}(x-32)^3]$ so $T_3(33) = \frac{2060643}{1024000} = 2.01234668$.
   Error: Take $M = |f^{(4)}(31)| = 1.7\times10^{-6}$ and hence $|R_3(33)| \le 7.2\times10^{-8}$ (*i.e.* good to 7 decimal places).
   So $\sqrt[5]{33} = 2.01234668 \pm 1.7\times10^{-6}$

4. (a) -1 (b) $\frac{1}{10}$

5. (a) x-int: $x=0$ at $t=-3$, $x=-9$ at $t=0$; y-int: $y=0$ at $t=-3$, $y=18$ at $t=3$.
   $\frac{dy}{dx} = 1+\frac{3}{2t}$, $\frac{d^2y}{dx^2} = -\frac{3}{4t^3}$. HT at $t=-\frac{3}{2}$, $(-\frac{27}{4},-\frac{9}{4})$; VT at $t=0$, $(-9,0)$.
   Graph: See Fig 1b.
   (b) area $= \int_{-3}^{0}(t^2+3t)2t\,dt = \frac{27}{2}$ length $= \int_{-3}^{0}\sqrt{(2t)^2+(2t+3)^2}\,dt$

6. Graph: see Fig 2b
   area $= \frac{1}{2}\int_{2\pi/3}^{4\pi/3}(1+2\cos\theta)^2\,d\theta = \pi-\frac{3}{2}\sqrt{3}$
   length $= 2\int_{0}^{2\pi/3}\sqrt{4\sin^2\theta+(1+2\cos\theta)^2}\,d\theta$