



Calculus III (Maths 201-DDB)

Answers

1. $R = 4$, Int of conv: $[-6, 2)$

2. $\int_0^x \sin(t^2) dt = \frac{x^3}{3} - \frac{x^7}{7 \cdot 3!} + \frac{x^{11}}{11 \cdot 5!} - \frac{x^{15}}{15 \cdot 7!} \pm \dots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+3}}{(4n+3)(2n+1)!}$

$\int_0^{1/2} \sin(x^2) dx \simeq \frac{1}{24} - \frac{1}{5376} \simeq 0.041480654$ (to within $\pm 10^{-6}$)

since the $|\text{error}| \leq \frac{1}{11 \cdot 5! \cdot 2^{11}} < 3.70 \times 10^{-7}$

3. $\frac{x}{\sqrt{1-x^2}} = x + \frac{1}{2}x^3 + \frac{1 \cdot 3}{2^2 \cdot 2!}x^5 + \frac{1 \cdot 3 \cdot 5}{2^3 \cdot 3!}x^7 + \dots = x + \sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^n \cdot n!} x^{2n+1}$

converges for $-1 < x < 1$.

4. $T_3(x) = 2 + \frac{1}{12}(x-8) - \frac{1}{288}(x-8)^2 + \frac{5}{20736}(x-8)^3$, so $\sqrt[3]{8.5} \simeq T_3(0.5) = 2.04082875$. For $|R_3(x)|$, we use the estimates: $M = \frac{80}{81}(7.5)^{-11/3} \simeq 6.1 \times 10^{-4}$, and $|R_3(x)| \leq \frac{M}{4!}(0.5)^4 \simeq 1.59 \times 10^{-6}$. So $\sqrt[3]{8.5} = 2.04082875 \pm 1.59 \times 10^{-6}$.

5. (a) $\frac{1}{2}$ (b) $\frac{1}{10}$

6. (a) x -int: $(0, 0)$ at $t = 0$; $(-9, 0)$ at $t = \pm\sqrt{3}$

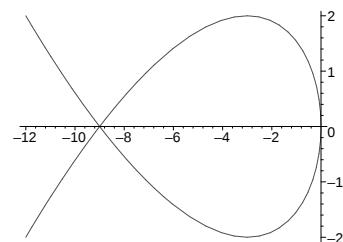
y -int: $(0, 0)$ again. $\frac{dy}{dx} = \frac{t^2-1}{2t}$ $\frac{d^2y}{dx^2} = \frac{t^2+1}{-12t^3}$

VT: at $(0, 0)$ ($t = 0$).

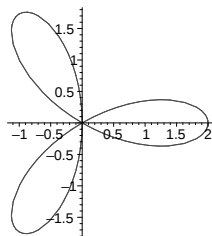
HT: at $(-3, -2)$ ($t = -1$) and $(-3, 2)$ ($t = 1$).

Graph at right: (orientation: start top left, go down right, around & up, finally back down to bottom left)

(b) length: $12\sqrt{3}$ area: $\frac{72}{5}\sqrt{3}$



7. Graphs: (a)



(b)

