Calculus III (Maths 201–DDB)

Answers

1. $\sum_{n=0}^{\infty}(-1)^n\frac{x^{4n+1}}{(4n+1)(2n+2)!} \simeq \frac{1}{4}-\frac{1}{3840}\pm\frac{1}{3317760} = 0.2497396 \pm 3\times 10^{-7}$

2. (a) $\sum_{n=0}^{\infty}(-1)^n x^{2n}$, IofC: $(-1,1)$ (b) $\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1}$, IofC: $[-1,1]$
   (c) $\pi/4 = \arctan 1 \simeq 1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}=\frac{263}{315}=0.83$ $(\pm\frac{1}{11}=\pm 0.09)$

3. $T_3(x)=2[1+\frac{1}{160}(x-32)-\frac{1}{12800}(x-32)^2+\frac{3}{2048000}(x-32)^3]$ so $T_3(33)=\frac{2060643}{1024000}=2.01234668$.
   Error: Take $M=|f^{(4)}(31)|=1.7\times 10^{-6}$ and hence $|R_3(33)|\le 7.2\times 10^{-8}$ (*i.e.* good to 7 decimal places).
   So $\sqrt[5]{33}=2.01234668\pm 1.7\times 10^{-6}$

4. (a) 1 (b) $\frac{1}{10}$

5. (a) x-int: $x=0$ at $t=-2$, $x=-4$ at $t=0$; y-int: $y=0$ at $t=-2$, $y=8$ at $t=2$.
   $\frac{dy}{dx}=1+\frac{1}{t}$, $\frac{d^2y}{dx^2}=-\frac{1}{2t^3}$. HT at $t=-1$, $(-3,-1)$; VT at $t=0$, $(-4,0)$.
   Graph: See Fig 1a.
   (b) area $=\int_{-2}^{0}(t^2+2t)2t\,dt=\frac{8}{3}$ length $=\int_{-2}^{0}\sqrt{(2t)^2+(2t+2)^2}\,dt$

6. Graph: see Fig 2a
   area $=\frac{1}{2}\int_{\pi/6}^{5\pi/6}(1-2\sin\theta)^2\,d\theta=\pi-\frac{3}{2}\sqrt{3}$
   length $=2\int_{-\pi/2}^{\pi/6}\sqrt{4\cos^2\theta+(1-2\sin\theta)^2}\,d\theta$