Instructor: Dr. R.A.G. Seely

Test 1
(version A)

Calculus III (Maths 201–DDB)

(Marks)

Note: Justify all your answers — don't make me guess your thoughts!

(2×3) 1. Determine whether the following series converge absolutely, converge conditionally, or diverge. Make clear what tests you are using.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^3}$ (b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n (n!)^2}{(2n+1)!}$

(5) 2. What are the radius and interval of convergence of the power series $\displaystyle\sum_{n=0}^{\infty} \frac{(x-2)^n}{2^{2n}(n+1)}$?

(7) 3. What is the Maclaurin series for $f(x) = x^3 \mathrm{e}^{-x^2}$? Use this series to estimate the value of $\displaystyle\int_0^{1/2} t^3 \mathrm{e}^{-t^2}\, dt$ correct within $\pm 10^{-4}$. Justify your estimate.

(6) 4. Use the Binomial Theorem to find the Taylor series around $x = 100$ for $f(x) = \sqrt{x}$. Estimate the possible error in using the second degree Taylor polynomial $T_2(x)$ to approximate $\sqrt{x}$ for $90 \le x \le 110$. (Hint: you will need to use Taylor's inequality. Also notice $x = 100 + (x - 100)$.)

5. Consider the curve given by the following parametric equations: $\begin{cases} x = t^2 \\ y = 9t - t^3 \end{cases}$

(6) (a) Find the x and y intercepts. Find $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$, and all points with horizontal and vertical tangents. Find all points of inflection (where the curve changes concavity). Sketch the graph, showing all these points. Indicate the direction of increasing t (the "orientation").

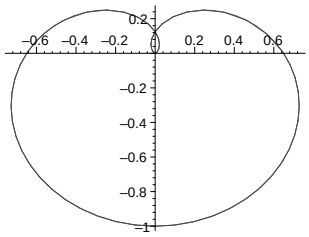
(5) (b) Find the area of the loop created by the curve.

(3) (c) Set up (but don't evaluate) the integral needed to calculate the arc length of the loop.

(7) 6. Sketch the graph of $r = 1 + \sin\theta$. Calculate the area enclosed by this graph.

(5) 7. The graph of $r = \sin^3\left(\frac{\theta}{3}\right)$ is sketched at the right.

Find the (arc) length of this curve.
(Hint: determine what values of θ are necessary to sketch out the entire curve.)



(Total: 50)