Instructor: Dr. R.A.G. Seely — Test 1

Calculus III (Maths 201–DDB)

(Marks)

Note: Justify all your answers — don't make me guess your thoughts!

(5) 1. What are the radius and interval of convergence of the power series $\displaystyle\sum_{n=0}^{\infty} \frac{(x+2)^n}{2^{2n}\sqrt[3]{n+1}}$?

(7) 2. Construct a power series for $\displaystyle\int_0^x \sin(t^2)\,dt$; use this series to approximate $\displaystyle\int_0^{1/2} \sin(x^2)\,dx$ to within $\pm 10^{-6}$. Justify your error estimate.

(6) 3. Use the Binomial Theorem to find the Maclaurin series for the function $f(x) = \dfrac{x}{\sqrt{1-x^2}}$. What is the interval of convergence for this series?

(7) 4. What is the third degree Taylor polynomial $T_3(x)$ for the function $f(x) = \sqrt[3]{x}$ centered at $x = 8$? Use $T_3(x)$ to approximate $\sqrt[3]{8.5}$. Use Taylor's inequality to estimate the possible error of this approximation.

5. Two "quickies" *(Don't spend a lot of time on these!)*:

(3) (a) Use a known power series (one of the basic ones you have learned about) to evaluate the sum of the following series:

$$1 - \ln 2 + \frac{(\ln 2)^2}{2!} - \frac{(\ln 2)^3}{3!} + \frac{(\ln 2)^4}{4!} - \frac{(\ln 2)^5}{5!} \pm \cdots$$

Give your answer in simplified exact form (not a decimal, please).

(2) (b) Suppose $f(x) = \displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (x+3)^n}{n(n!)}$; find $f^{(10)}(-3)$ (without calculation).

6. Consider the curve given by the following parametric equations: $\begin{cases} x = -3t^2 \\ y = 3t - t^3 \end{cases}$

(6) (a) Find the x and y intercepts. Find $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$, and all points with horizontal and vertical tangents. Sketch the graph, showing all these points. Indicate the direction of increasing t (the "orientation").

(8) (b) The curve above forms a loop; find

i. the arc length of the loop and

ii. the area bounded by the loop.

(Hint: don't be intimidated by the first integral—it will work out easily once correctly simplified.)

(6) 7. Draw rough sketches of the graphs of the following polar curves. *(Hint: in each case, first find where $r = 0$ and so where the graph "flips" across the origin.)*

(a) $r = 2\cos 3\theta$

(b) $r = 1 + 2\sin\theta$

(Total: 50)