Assignment 5 Answers

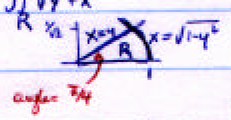
(1) 

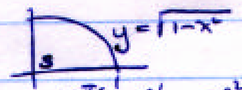
$$\int_0^1\int_0^x \cos x^2\,dy\,dx = \int_0^1 x\cos x^2\,dx = \tfrac12 \sin x^2\Big]_0^1 = \tfrac12\sin(1) = .4207$$

(2)(a)
$$= \int_0^4\int_0^{\sqrt x} \frac{y^3}{\sqrt{y^4+x^3}}\,dy\,dx = \frac12\int_0^4 \sqrt{y^4+x^3}\Big]_0^{\sqrt x}dx = \frac12\int_0^4\left(\sqrt{x^2+x^3}-x^{3/2}\right)dx = \frac12\int_0^4 x\sqrt{1+x}\,dx - \frac12\int_0^4 x^{3/2}\,dx$$
$$= \frac12\left(\frac25(1+x)^{5/2}-\frac23(1+x)^{3/2}\right]_0^4 - \frac{32}{5} = \frac{10\sqrt5}{3}+\frac{2}{15}-\frac{32}{5} = \frac{10\sqrt5}{3}-\frac{94}{15} = 1.1869$$

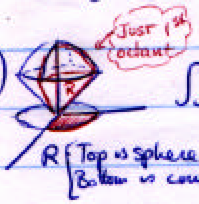
$u=1+x$, $du=dx$, $x=u-1$

(b)
$$= \iint_R \frac{y^2}{\sqrt{y^2+x^2}}\,dA = \int_0^{\pi/4}\int_0^1 \frac{r^2\sin^2\theta}{r}\,r\,dr\,d\theta = \int_0^{\pi/4}\frac13\sin^2\theta\,d\theta = \frac16\int_0^{\pi/4}(1-\cos2\theta)\,d\theta = \frac{\pi}{24}-\frac{1}{12}$$
$$\approx 0.0476$$





(c)
$$= \iint_S e^{-(x^2+y^2)}\,dA = \int_0^{\pi/2}\int_0^1 e^{-r^2}r\,dr\,d\theta = \frac{\pi}{2}\cdot\left(-\frac12 e^{-r^2}\Big]_0^1\right) = \frac{\pi}{4}\left(1-\frac1e\right) = .496$$

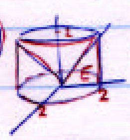
(d) 

$$\iiint_R z\,dz\,dx\,dy = \int_0^{\pi/2}\int_0^{\pi/4}\int_0^{\sqrt8}\rho^3\cos\varphi\sin\varphi\,d\rho\,d\varphi\,d\theta = \frac{\pi}{2}\cdot\int_0^{\pi/4}\cos\varphi\sin\varphi\,d\varphi\cdot\int_0^{\sqrt8}\rho^3\,d\rho$$
$$= \frac{\pi}{2}\cdot\frac14\cdot16 = 2\pi$$

R {Top is sphere $x^2+y^2+z^2=8$; Bottom is cone $z=\sqrt{x^2+y^2}$}

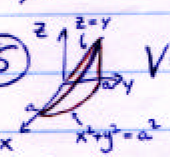
[N.B. cylindrical coords are possible too: $=\int_0^{\pi/2}\int_0^2\int_r^{\sqrt{8-r^2}} rz\,dz\,dr\,d\theta$]

(3)
$$= \int_0^4\int_0^{\sqrt y} xe^{y^2}\,dx\,dy = \frac12\int_0^4 ye^{y^2}\,dy = \frac14 e^{y^2}\Big]_0^4 = \frac14(e^{16}-1) = 2221527.38$$

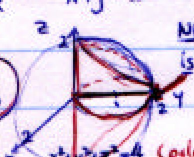
(4) 

$E\begin{cases}0\le\theta\le2\pi\\ \frac{\pi}{4}\le\varphi\le\pi/2\\ 0\le\rho\le2\csc\varphi\end{cases}$ $\quad \frac{2}{\rho}=\sin\varphi$

$$\iiint \frac{1}{x^2+y^2+z^2}\,dV = \int_0^{2\pi}\int_{\pi/4}^{\pi/2}\int_0^{2\csc\varphi}\sin\varphi\,d\rho\,d\varphi\,d\theta = 2\pi\int_{\pi/4}^{\pi/2}2\csc\varphi\sin\varphi\,d\varphi$$
$$= 4\pi\int_{\pi/4}^{\pi/2}d\varphi = \pi^2$$

(5) 

$$V=\int_0^{\pi/2}\int_0^a r^2\sin\theta\,dr\,d\theta = \int_0^{\pi/2}\sin\theta\,d\theta\cdot\int_0^a r^2\,dr = a^3/3$$

(6) 

NB $x^2+y^2-2y=0$ is $x^2+(y-1)^2=1$ (polar: $r=2\sin\theta$)

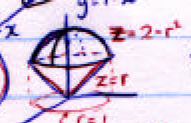
$$V=4\int_0^{\pi/2}\int_0^{2\sin\theta}\int_0^{\sqrt{4-r^2}} r\,dz\,dr\,d\theta = 4\int_0^{\pi/2}\int_0^{2\sin\theta}r\sqrt{4-r^2}\,dr\,d\theta$$
$$= 4\int_0^{\pi/2}\left(\frac83-\frac83\cos^3\theta\right)d\theta = \frac{16\pi}{3}-\frac{64}{9}$$
$$= 9.644$$

(Use symmetry to get the "bottom-back" halves - hence the factor 4)

(7) 

$$V=\int_0^1\int_0^{1-x}\int_0^{\cos(\frac{\pi x}{2})}dz\,dy\,dx = \int_0^1\int_0^{1-x}\cos\left(\frac{\pi x}{2}\right)dy\,dx = \int_0^1(1-x)\cos\left(\frac{\pi x}{2}\right)dx = \frac{4}{\pi^2}$$

[Hint: use ∫ by parts for $\int x\cos\frac{\pi x}{2}\,dx$]

(8) 

$$V=\int_0^{2\pi}\int_0^1(2-r^2-r)\,r\,dr\,d\theta = \frac{5\pi}{6}$$

(9) 

$$V=\int_0^{2\pi}\int_0^2(8-r^2-r^2)\,r\,dr\,d\theta = 16\pi$$

(10)(a)
$$=2\int_0^{\pi/6}\int_0^{4\sin\theta}r\,dr\,d\theta + 2\int_{\pi/6}^{\pi/2}\int_0^{4-4\sin\theta}r\,dr\,d\theta = 2\iint_R r\,dr\,d\theta$$ where R is 

(I'm using symmetry here, since there is a similar region in QII)