

Some Natural Deductions

Construct derivations for the following entailments:

1. $(A \rightarrow B) \rightarrow C \vdash B \rightarrow C$
2. $A \rightarrow B \vdash \neg(A \wedge \neg B)$
3. $\neg(A \vee B) \vdash \neg A \wedge \neg B$
4. $(A \rightarrow B) \vee (A \rightarrow C) \vdash A \rightarrow B \vee C$
5. $A \vee B \rightarrow C \vdash (A \rightarrow C) \wedge (B \rightarrow C)$
6. $A \vee B \vdash \neg B \rightarrow (C \rightarrow A)$

Translate the following (*i.e.* check my translations to see if you could duplicate them!), and construct derivations for these arguments.

1. If life is a carnival, then I'm a clown or a trapeze artist. But life isn't a carnival if there aren't any balloons, and there aren't any balloons if I'm a clown. So, if life is a carnival, then I'm a trapeze artist. ($L \rightarrow C \vee T, \neg B \rightarrow \neg L, C \rightarrow \neg B \vdash L \rightarrow T$.)
2. Albert is a Liberal only if Bruce or Carol is. If Bruce is a Liberal, so are Deirdre and Ethel. If Deirdre is a Liberal, then Ethel is a Liberal only if Freda is; but Freda and Albert aren't both Liberals. So Albert is a Liberal only if Carol is.
($A \rightarrow B \vee C, B \rightarrow D \wedge E, D \rightarrow (E \rightarrow F), \neg(F \wedge A) \vdash A \rightarrow C$)

Some Natural Deductions

Construct derivations for the following entailments:

1. $(A \rightarrow B) \rightarrow C \vdash B \rightarrow C$
2. $A \rightarrow B \vdash \neg(A \wedge \neg B)$
3. $\neg(A \vee B) \vdash \neg A \wedge \neg B$
4. $(A \rightarrow B) \vee (A \rightarrow C) \vdash A \rightarrow B \vee C$
5. $A \vee B \rightarrow C \vdash (A \rightarrow C) \wedge (B \rightarrow C)$
6. $A \vee B \vdash \neg B \rightarrow (C \rightarrow A)$

Translate the following (*i.e.* check my translations to see if you could duplicate them!), and construct derivations for these arguments.

1. If life is a carnival, then I'm a clown or a trapeze artist. But life isn't a carnival if there aren't any balloons, and there aren't any balloons if I'm a clown. So, if life is a carnival, then I'm a trapeze artist. ($L \rightarrow C \vee T, \neg B \rightarrow \neg L, C \rightarrow \neg B \vdash L \rightarrow T$.)
2. Albert is a Liberal only if Bruce or Carol is. If Bruce is a Liberal, so are Deirdre and Ethel. If Deirdre is a Liberal, then Ethel is a Liberal only if Freda is; but Freda and Albert aren't both Liberals. So Albert is a Liberal only if Carol is.
($A \rightarrow B \vee C, B \rightarrow D \wedge E, D \rightarrow (E \rightarrow F), \neg(F \wedge A) \vdash A \rightarrow C$)

The derivations

1. $(A \rightarrow B) \rightarrow C \vdash B \rightarrow C$

1	$(A \rightarrow B) \rightarrow C$				
2			B		
3				A	
4				B	(R), 2
5			$A \rightarrow B$		$(\rightarrow I), 3-4$
6			C		$(\rightarrow E), 1, 5$
7		$B \rightarrow C$			$(\rightarrow I), 2-6$

2. $A \rightarrow B \vdash \neg(A \wedge \neg B)$

1	$A \rightarrow B$	
2	$A \wedge \neg B$	
3	A	$(\wedge E), 2$
4	$\neg B$	$(\wedge E), 2$
5	B	$(\rightarrow E), 1, 3$
6	$\perp$	$(\neg E), 4, 5$
7	$\neg(A \wedge \neg B)$	$(\neg I), 2-6$

3. $\neg(A \vee B) \vdash \neg A \wedge \neg B$

1		$\neg(A \vee B)$	
2			A
3			$A \vee B$ $(\vee I), 2$
4			$\perp$ $(\neg E), 1, 3$
5		$\neg A$	$(\neg I), 2-4$
6			B
7			$A \vee B$ $(\vee I), 6$
8			$\perp$ $(\neg E), 1, 7$
9		$\neg B$	$(\neg I), 6-8$
10		$\neg A \wedge \neg B$	$(\wedge I), 5, 9$

4. $(A \rightarrow B) \vee (A \rightarrow C) \vdash A \rightarrow B \vee C$

1		$(A \rightarrow B) \vee (A \rightarrow C)$			
2			A		
3			$(A \rightarrow B) \vee (A \rightarrow C)$	(R), 1	
4				$A \rightarrow B$	
5				B	($\rightarrow$ E), 2, 4
6				$B \vee C$	($\vee$ I), 5
7				$A \rightarrow C$	
8				C	($\rightarrow$ E), 2, 7
9				$B \vee C$	($\vee$ I), 8
10			$B \vee C$	($\vee$ E), 3, 4-6, 7-9	
11		$A \rightarrow B \vee C$		($\rightarrow$ I), 2-10	

5. $A \vee B \rightarrow C \vdash (A \rightarrow C) \wedge (B \rightarrow C)$

1		$A \vee B \rightarrow C$		
2			A	
3			$A \vee B$	$(\vee\text{I}), 2$
4			C	$(\rightarrow\text{E}), 1, 3$
5		$A \rightarrow C$		$(\rightarrow\text{I}), 2\text{--}4$
6			B	
7			$A \vee B$	$(\vee\text{I}), 6$
8			C	$(\rightarrow\text{E}), 1, 7$
9		$B \rightarrow C$		$(\rightarrow\text{I}), 6\text{--}8$
10		$(A \rightarrow C) \wedge (B \rightarrow C)$		$(\wedge\text{I}), 5, 9$

6. $A \vee B \vdash \neg B \rightarrow (C \rightarrow A)$

1		$A \vee B$	
2			$\neg B$
3			
4			
5			
6			$C \rightarrow A$
7			
8			$\perp$
9			$C \rightarrow A$
10			$C \rightarrow A$
11		$\neg B \rightarrow (C \rightarrow A)$	

(R), 3

($\rightarrow$ I), 4-5

($\neg$ E), 2, 7

($\perp$ E), 8

($\vee$ E), 1, 3-6, 7-9

($\rightarrow$ I), 2-9

And now the wordy ones:

1.

1		$L \rightarrow C \vee T$	
2		$\neg B \rightarrow \neg L$	
3		$C \rightarrow \neg B$	
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			

2.

1		$A \rightarrow B \vee C$	
2		$B \rightarrow D \wedge E$	
3		$D \rightarrow (E \rightarrow F)$	
4		$\neg(F \wedge A)$	
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			