Maths & Logic (360-124)

The Answers

1. (a) 10202221_3 (b) 1000011010_2 (c) 56455_7 (d) 517_9

2. (a) 725 (b) 526 (c) 416 (d) 421

3. (a) $\frac{1}{1\cdot 2} = \frac{1}{2}$; $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \cdots + \frac{1}{n(n+1)} + \frac{1}{(n+1)(n+2)} = \frac{n}{n+1} + \frac{1}{(n+1)(n+2)} = \frac{n(n+2)+1}{(n+1)(n+2)} = \frac{(n+1)^2}{(n+1)(n+2)} = \frac{n+1}{n+2}$.

   (b) $5|8^1 - 3^1 = 5$; $8^{n+1} - 3^{n+1} = 8\cdot 8^n - 3^{n+1} = (5+3)8^n - 3\cdot 3^n = 5\cdot 8^n + 3\cdot(8^n - 3^n)$ and 5 divides each of these terms.

   (c) $1 = 1^2$; $1+3+5+\cdots+(2n+1) = 1+3+5+\cdots+(2n-1)+(2n+1) = n^2+(2n+1) = (n+1)^2$.

4. Given $bq + r = dx$, $b = dy$ for some x, y, then $r = (bq + r) - bq = dx - dyq = d(x - yq)$ so $d \mid r$.

5. $2^3 3^3 57$

6. $1701 = 3^5 \cdot 7$, so the factors are: $1, 3, 9, 27, 81, 243, 7, 21, 63, 189, 567, 1701$.

7. 7×103

8. 647 (the number is prime)

9. $11! + 1$, or $2\cdot 3\cdot 5\cdot 7\cdot 11 + 1$. (You want 10 consecutive composites in all, so **start** at $11! + 2$ or the similar product using only primes. That means the number before the list is $11! + 1$ or the corresponding primes-only expression.)

10. Any composite number will do — express it as a product of smaller numbers to show it's not special. *E.g.* 24 is not special, since $24 \mid 4 \times 6$ but 24 does not divide either 4 or 6.

11. As done in the text for $\sqrt{3}$ on p.200, or for $\sqrt{2}$ on p.172.