Maths & Logic (360-124)

Note: all these problems have been done in class or are in the notes, possibly with different letters used — so you can find the answers if you search hard enough!

1. Given the following derivations, show that each is correct by filling in correct justifications (names of rules and line numbers, as required). (To illustrate what I want, I have done the first line for you.)

1	$A \to B \lor C$	
2	$B \to D \land E$	
3	$D \to (E \to F)$	
4	$\neg(F \land A)$	
5	A	
6	$B \lor C$	$(\to E), 1, 5$
7	B	
8	$D \land E$	
9	D	
10	E	
11	$E \to F$	
12	F	
13	$F \land A$	
14	$\bot$	
15	C	
16	C	
17	C	
18	C	
19	$A \to C$	

1	$B \to F$	
2	$\neg B \land \neg F \to H$	
3	$\neg H \land \neg F$	
4	$\neg H$	$(\land E), 3$
5	$\neg F$	
6	B	
7	F	
8	$\bot$	
9	$\neg B$	
10	$\neg B \land \neg F$	
11	H	
12	$\bot$	

2. Give correct derivations for each of the following valid arguments. Be sure to only use the basic "intro" and "elim" rules (or, if you do use a "derived rule", state it explicitly before you start your derivation, and if it isn't one we did in class, provide a derivation for your derived rule). Be sure to correctly justify each line in your derivation.

1	$p \to q$	
2	$p \to r$	
?	$p \to q \land r$	Find a proof

1	$(p \to r) \lor (q \to r)$	
?	$p \land q \to r$	Find a proof

3. Show the following is **not** valid by the method of tableaux. Use your tableau to give appropriate truth values to all "atoms" which invalidate the argument (*i.e.* so that the premises are true, but the conclusion is false).

1	$A \to (B \to C)$
2	$C \land D \to \neg E$
3	$\neg F \to D \land E$
?	$\neg C \to \neg E \land \neg F$

4. Show that the following arguments are valid, by any method you like: tableaux, truth tables, or formal derivations. Be sure to be precise, however, and use only formally valid methods (no "waffle"!)

Hint: the first one uses the $(\neg\neg E)$ rule.

1	$\neg(p \land \neg q)$
?	$p \to q$

1	$A \to B$
2	$(C \lor B) \land \neg B$
3	$C \to D$
?	$A \lor D$

5. Translate the following into a formal argument, and prove it is valid by any formal method you like. (The same "caution" applies to this problem as to the previous, however.)

> Unless the Vulcans leave the Federation and join the Romulans, the Klingons will attack the Romulans. If the Klingons attack the Romulans, the Romulans will surrender and join the Klingons to attack the Federation. If the Klingons and Romulans together attack the Federation, the Federation will be destroyed. Therefore, if the Vulcans remain in the Federation, it will be destroyed.

Use the following abbreviations: V = The Vulcans will leave the Federation; R = The Vulcans will join the Romulans; K = The Klingons will attack the Romulans; S = The Romulans will surrender; J = The Romulans will join the Klingons; A = The Romulans and Klingons will attack the Federation; D = The Federation will be destroyed.

6. And the essay ...