Algebraic Topology, problem list 7

Problem 1. Let M_g be the closed orientable surface of genus g with CW structure consisting of one vertex, $2g$ edges labelled a_j, b_j, and one 2-cell attached by the product of commutators $[a_1, b_1] \cdots [a_g, b_g]$. Compute the groups $H_i^{CW}(M_g)$.

Problem 2. Let X be obtained from $S^1 \vee S^1$ by attaching two 2-cells by the words $a^5 b^{-3}$ and $b^3 (ab)^{-2}$.

(i) Prove that $\widetilde{H}_i(X) = 0$ for all k.

(ii) Prove that X is not contractible. Hint: dodecahedron.

Problem 3. Compute the groups $H_i^{CW}(\mathbf{R}P^n)$ for $n = 1, 2, \ldots, \infty$.

Problem 4. Prove that if a finite CW complex is a $K(G, 1)$, then G does not have a finite subgroup of even order.

Problem 5. Let $f \colon X \to Y$ be a map between simply-connected CW complexes inducing isomorphisms on all H_i. Prove that f is a homotopy equivalence.