Algebraic Topology, problem list 5

Problem 1. Complete the proof of the long exact sequence theorem, i.e. prove that ∂ is well-defined and that the sequence is exact.

Problem 2. Let (X, A) be a CW pair. Prove that A has a neighborhood V in X that deformation retracts to A.

Problem 3. Prove that $\widetilde{H}_i(S^n) = 0$ for $i \neq n$. Hint: long exact sequence theorem and the corollary to the excision theorem.

Problem 4. Prove that if nonempty open sets $U \subset \mathbf{R}^m$ and $V \subset \mathbf{R}^n$ are homeomorphic, then $m = n$. Hint: a homeomorphism $h\colon U \to V$ would induce isomorphisms $H_k(U, U - x) \to H_k(V, V - h(x))$.