Algebraic Topology, problem list 4

Problem 1. Prove that $\pi_4(S^3)$ is a cyclic group.

Freudenthal theorem is a special case of the following, which you are allowed to use.

Theorem. *Let X be a* CW *complex decomposed as a union of subcomplexes A, B with nonempty connected intersection $C = A \cap B$. Suppose that (A, C) is m-connected and (B, C) is n-connected, with $m, n \geq 0$. Then the map $\pi_i(A, C) \to \pi_i(X, B)$ induced by inclusion is an isomorphism for $i < m + n$ and a surjection for $i = m + n$.*

Problem 2. Explain how Freudenthal theorem follows from the above theorem.

Problem 3. Let (A, C) be an m-connected CW pair with with $(n-1)$-connected C. Prove that $\pi_i(A, C) \to \pi_i(A/C)$ is an isomorphism for $i < m+n$ and a surjection for $i = m + n$. Hint: take B to be the cone over C and $X = A \cup B$.

Problem 4. Let $n \geq 2$. Suppose that A is obtained from a bouquet of spheres S^n by attaching $(n + 1)$-cells via attaching maps representing prescribed elements $g \in \pi_n(\bigvee S^n)$.

(i) Compute $\pi_n(A)$ in terms of the g's.

(ii) Construct $K(G, n)$ for any abelian G.

Problem 5. Compute $\pi_3(S^2 \vee S^2)$ and describe its generators. Hint: long exact sequence of $(S^2 \times S^2, S^2 \vee S^2)$.