Algebraic Topology, problem list 3

Problem 1. Let $f\colon X \to Y$ be a cellular map between CW complexes and let M_f be the mapping cylinder $X \times I \sqcup Y/(x,1) \sim f(x)$. Prove that $(M_f, X\times 0)$ is a CW pair.

Problem 2. The *Warsaw circle* is the closed subspace of $\mathbf{R}^2$ consisting of the graph of the function $y = \sin\frac{1}{x}$ for $x \in (0,1]$, the segment $[-1,1]$ in the y-axis and an arc connecting these two pieces. Prove that the Warsaw circle is not homotopy equivalent to a CW complex by computing its homotopy groups.

Problem 3. Let X be a CW complex that is a direct limit of its subcomplexes $X_1 \subset X_2 \subset \cdots$. Assume that each inclusion $X_i \subset X_{i+1}$ is a contractible map. Prove that X is contractible.

Problem 4. Construct $K(\mathbf{Z}_3, 1)$. Hint: find compatible Z_3 actions on all S^{2n+1}.

Problem 5. Let X be a CW complex and let Y be a $K(G,1)$. Prove that each homomorphism $\pi_1(X,x_0) \to \pi_1(Y,y_0)$ is induced by a continuous map $(X,x_0) \to (Y,y_0)$, which is unique up to a basepoint preserving homotopy.