Algebraic Topology, problem list 2

Problem 1. Find a CW complex structure on

(i) S^∞,

(ii) $\mathbf{R}P^\infty$,

(iii) $\mathbf{C}P^\infty$.

Problem 2. Let (X, A) be a CW pair with A contractible. Prove that $X \to X/A$ is a homotopy equivalence. Hint: use the homotopy extension property.

Problem 3. Compute $\pi_i(\mathbf{C}P^n)$ for all $i \leq 2n$.

Problem 4. (i) Prove that every map $f\colon (X, A) \to (Y, B)$ of CW pairs is homotopic through maps of the same form to a cellular map.

(ii) Deduce that if all cells in $X - A$ have dimension $> i$, then $\pi_i(X, A) = 0$.

(iii) Deduce that the inclusion of the n-skeleton $X^n \subset X$ induces isomorphisms on π_i for $i < n$.

Problem 5. Let X be the bouquet of k spheres S^n. Prove that for $i < 2n-1$ we have $\pi_i(X) = \pi_i(S^n)^k$.