```
> ## Assignment #8 Mast 334/ Math 354 Solutions
## Problem 4 page 494
## data:
xx:=[0,0.15,0.31,0.5,0.6,0.75];
yy:=[1.0,1.004,1.031,1.117,1.223,1.422];
xy:=seq([xx[i],yy[i]],i=1..6):
plotdatapoints:=(POINTS(xy,SYMBOL(CIRCLE),COLOR(RGB,1,0,0))
):
PLOT(plotdatapoints);
```
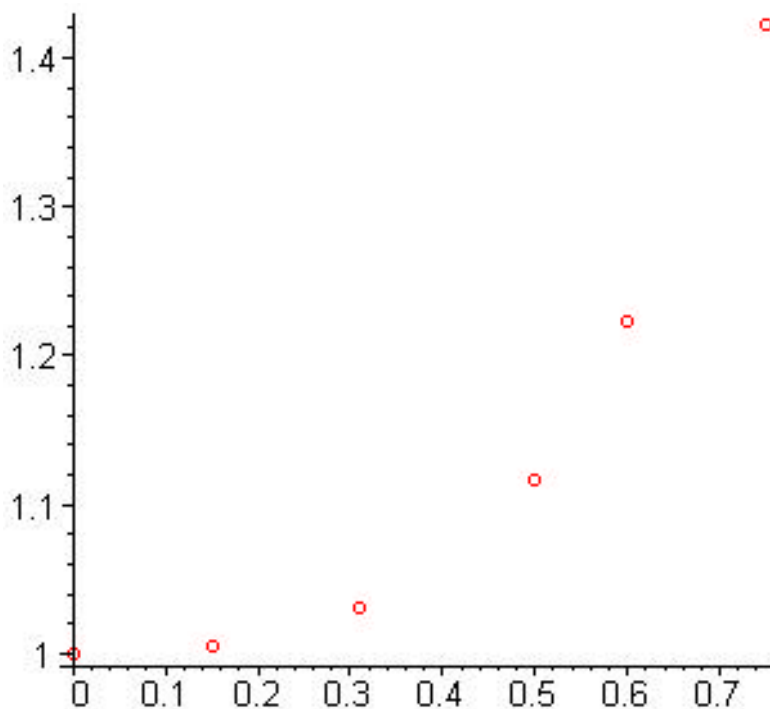
$$xx := [0, 0.15, 0.31, 0.5, 0.6, 0.75]$$

$$yy := [1.0, 1.004, 1.031, 1.117, 1.223, 1.422]$$



```
> # Linear Least Squares: f1(x)=ax+b:
# equations:
#   a*sum(x_i)     + b*sum(1)     = sum(y_i)
#   a*sum(x_i)^2   + b*sum(x_i)   = sum(x_iy_i)
sum1:=sum(1,i=1..6);
sumx1:=sum(xx[i],i=1..6);
sumx2:=sum(xx[i]^2,i=1..6);
sumy1:=sum(yy[i],i=1..6);
sumxy:=sum(xx[i]*yy[i],i=1..6);
```

$$sum1 := 6$$
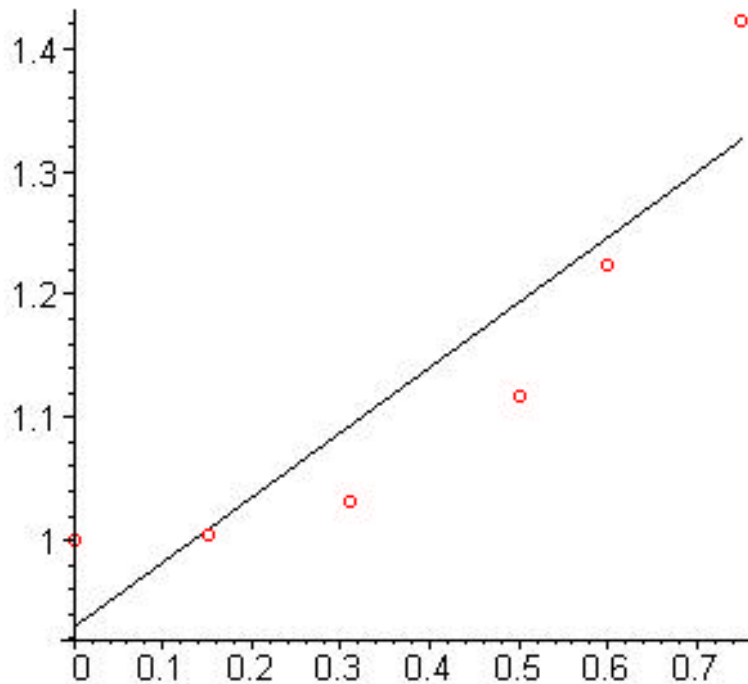
$$sumx1 := 2.31$$

$$sumx2 := 1.2911$$

$$sumy1 := 6.797$$

$$sumxy := 2.82901$$

```
> EQ1:=a*sumx1  +  b*sum1    =sumy1:
  EQ2:=a*sumx2  +  b*sumx1  =sumxy:
  sol:=solve({EQ1,EQ2},{a,b});
```
$$sol := \{ b = 0.9295140427, a = 0.5281020535 \}$$

```
> assign(sol):
> f1:=x->a*x+b:
  dd:=(xx[6]-xx[1])/1000.0:
  f1points:=[seq([xx[1]+i*dd,f1(xx[1]+i*dd)],i=0..1000)]:
   PLOT(plotdatapoints,CURVES(f1points));
  err:=sum((f1(xx[i])-yy[i])^2,i=1..6);
```



$$err := 0.02456606118$$

```
> # Quadratic Least Squares: f2(x)=ax^2+bx+c:
  # equations:
  #  a*sum(x_i)^2    + b*sum(x_i)    + c*sum(1)   = sum(y_i)
  #  a*sum(x_i)^3    + b*sum(x_i)^2  + c*sum(x_i) =
  sum(x_iy_i)
  #  a*sum(x_i)^4    + b*sum(x_i)^3  + c*sum(x_i)^2 =
  sum(x_i^2y_i)
  # additional sums we need:
  sumx3:=sum(xx[i]^3,i=1..6);
  sumx4:=sum(xx[i]^4,i=1..6);
  sumx2y:=sum(xx[i]^2*yy[i],i=1..6);
```
$$sumx3 := 0.796041$$

$$sumx4 := 0.51824771$$

$$sumx2y := 1.6410741$$

```
> a:='a':b:='b':c:='c':# making a, b, c variable again
EQ1:=a*sumx2  +  b*sumx1  +  c*sum1     =sumy1:
EQ2:=a*sumx3  +  b*sumx2  +  c*sumx1    =sumxy:
EQ3:=a*sumx4  +  b*sumx3  +  c*sumx2    =sumx2y:
sol2:=solve({EQ1,EQ2,EQ3},{a,b,c});
```
$$sol2 := \{\, b = -0.3256987507,\ c = 1.011340993,\ a = 1.147330305 \,\}$$

```
> assign(sol2):
> f2:=x->a*x^2+b*x+c:
dd:=(xx[6]-xx[1])/1000.0:
f2points:=[seq([xx[1]+i*dd,f2(xx[1]+i*dd)],i=0..1000)]:
 PLOT(plotdatapoints,CURVES(f2points));
err:=sum((f2(xx[i])-yy[i])^2,i=1..6);
```



$$err := 0.000945246274(\,$$
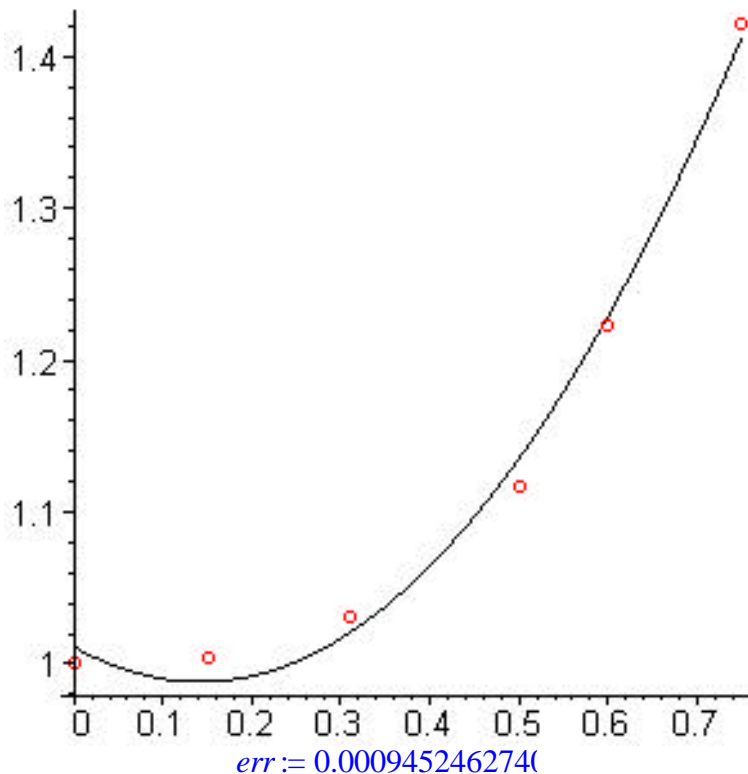
```
> # Cubic Least Squares: f2(x)=ax^3+bx^2+cx+d:
# equations:
#  a*sum(x_i)^3    + b*sum(x_i)^2    +  c*sum(x_i) +
d*sum(1)    = sum(y_i)
#  a*sum(x_i)^4    + b*sum(x_i)^3  + c*sum(x_i)^2 +
d*sum(x_i) = sum(x_iy_i)
#  a*sum(x_i)^5    + b*sum(x_i)^4  + c*sum(x_i)^3 +
d*sum(x_i)^2= sum(x_i^2y_i)
#  a*sum(x_i)^6    + b*sum(x_i)^5  + c*sum(x_i)^4 +
d*sum(x_i)^3= sum(x_i^3y_i)
```
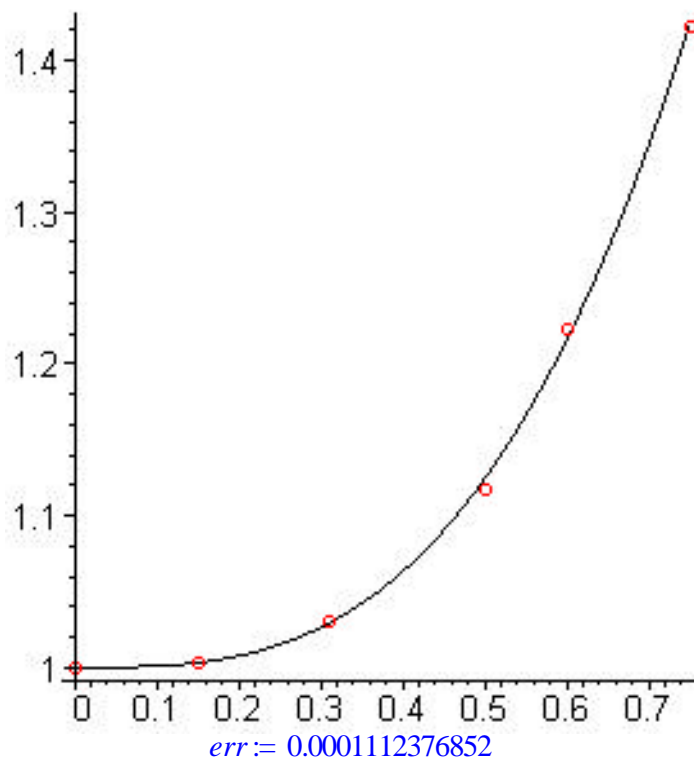
```
# additional sums we need:
sumx5:=sum(xx[i]^5,i=1..6);
sumx6:=sum(xx[i]^6,i=1..6);
sumx3y:=sum(xx[i]^3*yy[i],i=1..6);
```

$$sumx5 := 0.3492535401$$

$$sumx6 := 0.2411584099$$

$$sumx3y := 1.037802271$$

```
>a:='a':b:='b':c:='c':d:='d':# making a, b, c, d variable
again
EQ1:=a*sumx3  +  b*sumx2  +  c*sumx1 + d*sum1  =sumy1:
EQ2:=a*sumx4  +  b*sumx3  +  c*sumx2 + d*sumx1 =sumxy:
EQ3:=a*sumx5  +  b*sumx4  +  c*sumx3 + d*sumx2  =sumx2y:
EQ4:=a*sumx6  +  b*sumx5  +  c*sumx4 + d*sumx3 =sumx3y:
sol3:=solve({EQ1,EQ2,EQ3,EQ4},{a,b,c,d});
```

$$sol3 := \{ d = 1.000439805 , c = -0.001540973487 , b = -0.01150571942 , a = 1.021022604 \}$$

```
>assign(sol3):
>f3:=x->a*x^3+b*x^2+c*x+d:
dd:=(xx[6]-xx[1])/1000.0:
f3points:=[seq([xx[1]+i*dd,f3(xx[1]+i*dd)],i=0..1000)]:
 PLOT(plotdatapoints,CURVES(f3points));
err:=sum((f3(xx[i])-yy[i])^2,i=1..6);
```



$$err := 0.0001112376852$$

```
>
```