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> ### Assignment #5, Mast 334/ Math 354 Solutions
>
> #Problem 3c page 119
x0:=0.1:   f0:=-0.62049958:## This value was incosistent
                              ## with Problem 7c I have
changed it
x1:=0.2:   f1:=-0.28398668:
x2:=0.3:   f2:=0.00660095:
x3:=0.4:   f3:=0.24842440:
# we are looking for interpolations of value f(0.25):
> #Linear using x1 and x2:
#Building blocks:
L1x1:=x->(x-x2)/(x1-x2);
L1x2:=x->(x-x1)/(x2-x1);
# Linear interpolating polynomial:
L1:=x->f1*L1x1(x)+f2*L1x2(x);
# value at 0.25
L1(0.25);
```

$$L1x1 := x \rightarrow \frac{x - x2}{x1 - x2}$$

$$L1x2 := x \rightarrow \frac{x - x1}{x2 - x1}$$

$$L1 := x \rightarrow f1 \, \mathrm{L1x1}(x) + f2 \, \mathrm{L1x2}(x)$$

$$-0.13869286500000000($$

```
> #Quadratic using x0,x1 and x2:
#Building blocks:
L2x0:=x->((x-x1)*(x-x2))/((x0-x1)*(x0-x2));
L2x1:=x->((x-x0)*(x-x2))/((x1-x0)*(x1-x2));
L2x2:=x->((x-x0)*(x-x1))/((x2-x0)*(x2-x1));
# Quadratic interpolating polynomial:
L2:=x->f0*L2x0(x)+f1*L2x1(x)+f2*L2x2(x);
# value at 0.25
L2(0.25);
```

$$L2x0 := x \rightarrow \frac{(x - x1)\,(x - x2)}{(x0 - x1)\,(x0 - x2)}$$

$$L2x1 := x \rightarrow \frac{(x - x0)\,(x - x2)}{(x1 - x0)\,(x1 - x2)}$$

$$L2x2 := x \rightarrow \frac{(x - x0)\,(x - x1)}{(x2 - x0)\,(x2 - x1)}$$

$$L2 := x \rightarrow f0 \, \mathrm{L2x0}(x) + f1 \, \mathrm{L2x1}(x) + f2 \, \mathrm{L2x2}(x)$$

$$-0.13295220625000000000$$

```
> #Quadratic using x1 , x2 and x3:
#Building blocks:
L2x3:=x->((x-x1)*(x-x2))/((x3-x1)*(x3-x2));
L2x1a:=x->((x-x3)*(x-x2))/((x1-x3)*(x1-x2));# These blocks
are different then above
L2x2a:=x->((x-x3)*(x-x1))/((x2-x3)*(x2-x1));#  -----------
"------------
# Quadratic interpolating polynomial:
L2a:=x->f3*L2x3(x)+f1*L2x1(x)+f2*L2x2(x);
# value at 0.25
L2a(0.25);
```

$$L2x3 := x \to \frac{(x - x1)\,(x - x2)}{(x3 - x1)\,(x3 - x2)}$$

$$L2x1a := x \to \frac{(x - x3)\,(x - x2)}{(x1 - x3)\,(x1 - x2)}$$

$$L2x2a := x \to \frac{(x - x3)\,(x - x1)}{(x2 - x3)\,(x2 - x1)}$$

$$L2a := x \to f3\ \mathrm{L2x3}(x) + f1\ \mathrm{L2x1}(x) + f2\ \mathrm{L2x2}(x)$$

-0.2415677037500000000

```
> #Cubic  using x0, x1 , x2 and x3:
#Building blocks:
L3x0:=x->((x-x1)*(x-x2)*(x-x3))/((x0-x1)*(x0-x2)*(x0-x3));
L3x1:=x->((x-x0)*(x-x3)*(x-x2))/((x1-x0)*(x1-x3)*(x1-x2));
L3x2:=x->((x-x0)*(x-x3)*(x-x1))/((x2-x0)*(x2-x3)*(x2-x1));
L3x3:=x->((x-x0)*(x-x2)*(x-x1))/((x3-x0)*(x3-x2)*(x3-x1));
# Quadratic interpolating polynomial:
L3:=x->f3*L3x3(x)+f1*L3x1(x)+f2*L3x2(x)+f0*L3x0(x);
# value at 0.25
L3(0.25);
```

$$L3x0 := x \to \frac{(x - x1)\,(x - x2)\,(x - x3)}{(x0 - x1)\,(x0 - x2)\,(x0 - x3)}$$

$$L3x1 := x \to \frac{(x - x0)\,(x - x3)\,(x - x2)}{(x1 - x0)\,(x1 - x3)\,(x1 - x2)}$$

$$L3x2 := x \to \frac{(x - x0)\,(x - x3)\,(x - x1)}{(x2 - x0)\,(x2 - x3)\,(x2 - x1)}$$

$$L3x3 := x \to \frac{(x - x0)\,(x - x2)\,(x - x1)}{(x3 - x0)\,(x3 - x2)\,(x3 - x1)}$$

$$L3 := x \to f3\ \mathrm{L3x3}(x) + f1\ \mathrm{L3x1}(x) + f2\ \mathrm{L3x2}(x) + f0\ \mathrm{L3x0}(x)$$

-0.13277477437500000(

```
> ## Problem 7c page 119
> f:=x->x*cos(x)-2*x^2+3*x-1;
df:=D(f); # derivative
d2f:=D(df);   # 2-nd derivative
d3f:=D(d2f); # 3-rd derivative
```

$$f := x \to x\cos(x) - 2x^2 + 3x - 1$$

$$df := x \to \cos(x) - x\sin(x) - 4x + 3$$

$$d2f := x \to -2\sin(x) - x\cos(x) - 4$$

$$d3f := x \to -3\cos(x) + x\sin(x)$$

```
> ### Theoretical and actual errors:
# Linear case: |error|<= sup{|d2f(x)|:  x1 < x < x2}
*(1/2)*|x-x1|*|x-x2|
# We can find sup{|d2f(x)|:  x1 < x < x2} but we just use
an estimate:
# sup{|d2f(x)|:  x1 < x < x2}<
|2sin(x)|+|xcos(x)|+4<2+x2+4=6+x2=6.3
# Then the theoretical error is <
6.3*(1/2)*abs(0.25-x1)*abs(0.25-x2);
```

$$0.00787500000000000000$$

```
> # Actual linear error:
abs(L1(0.25)-f(0.25));
```

$$0.0059209704276611960 4$$

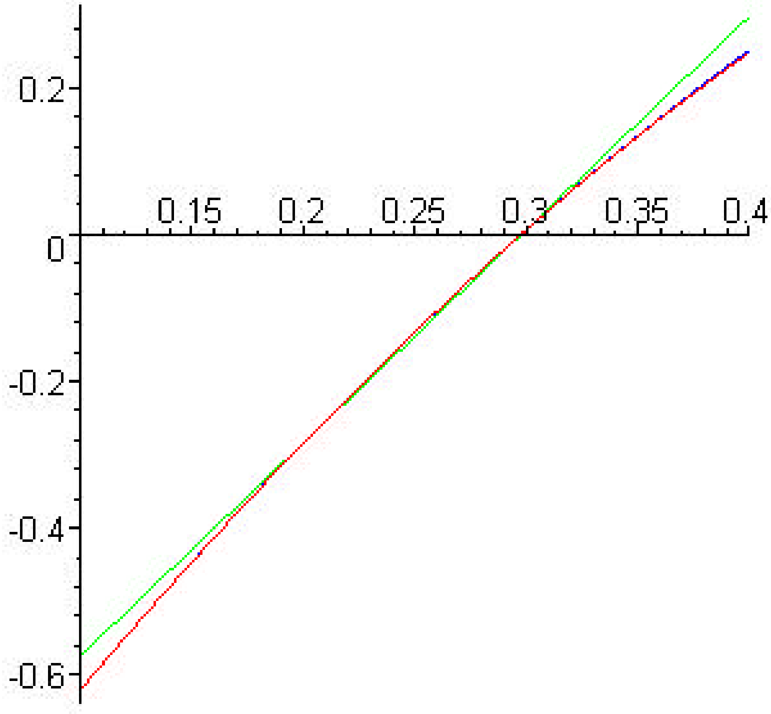
```
> # Quadratic case:we will do only the case using x0,x1,x2:
# |error|<= sup{|d3f(x)|:  x0 < x < x2} *(1/6)*|x-x0|*|x-
x1|*|x-x2|
# We can find sup{|d3f(x)|:  x0 < x < x2} but we just use
an estimate:
# sup{|d3f(x)|:  x0 < x < x2}< |3cos(x)|+|xsin(x)|<3+x2=3.3
# Then the theoretical error is <
3.3*(1/6)*abs(0.25-x0)*abs(0.25-x1)*abs(0.25-x2);
```

$$0.00020625000000000000($$

```
> # Actual quadratic error:
Digits:=20:
abs(L2(0.25)-f(0.25));
```

$$0.00018031167766119604$$

```
> plot([f,L1,L2],0.1..0.4,color=[red,green,blue]);
```




```
> f(0.1);## This value is given with an error in Problem 3c
in the book.
```

-0.6204995834721974233

```
> ### Problem 8 page 119
f:=x->sqrt(x-x^2);
```

$$f := x \to \sqrt{x - x^2}$$

```
> x0:=0;   f0:=f(x0);
x2:=1;   f2:=f(x2);
```

$$x0 := 0$$

$$f0 := 0$$

$$x2 := 1$$

$$f2 := 0$$

```
> ## Quadratic Lagrange interpolation with a variable x1=z:
# Building blocks:

L2x0:=x->((x-z)*(x-x2))/((x0-z)*(x0-x2));
L2z:=x->((x-x0)*(x-x2))/((z-x0)*(z-x2));
L2x2:=x->((x-x0)*(x-z))/((x2-x0)*(x2-z));
# Quadratic interpolating polynomial:
L2:=x->f0*L2x0(x)+f(z)*L2z(x)+f2*L2x2(x);
```

$$L2x0 := x \to \frac{(x-z)(x-x2)}{(x0-z)(x0-x2)}$$

$$L2z := x \to \frac{(x-x0)(x-x2)}{(z-x0)(z-x2)}$$

$$L2x2 := x \to \frac{(x-x0)(x-z)}{(x2-x0)(x2-z)}$$

$$L2 := x \to f0\ \mathrm{L2x0}(x) + \mathrm{f}(z)\ \mathrm{L2z}(x) + f2\ \mathrm{L2x2}(x)$$

```
> simplify(expand(L2(x)));
```

$$\frac{\sqrt{-z\,(z-1)}\,x\,(x-1)}{z\,(z-1)}$$

```
> # Maple did not see it somehow but we can see that:
L2:=x->(x*(1-x))/sqrt(z*(1-z));
```

$$L2 := x \to \frac{x\,(1-x)}{\sqrt{z\,(1-z)}}$$

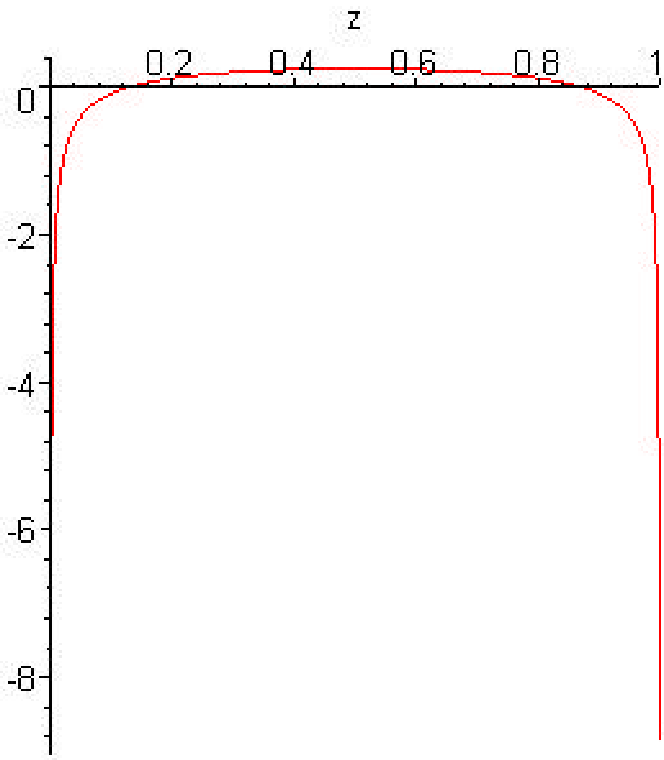
```
> ## Now we have to find the largest z in (0,1) for which
f(0.5)-L2(0.5)=-0.25:
g:=z->f(0.5)-L2(0.5)+0.25;
```

$$g := z \to \mathrm{f}(0.5) - \mathrm{L2}(0.5) + 0.25$$

```
> expand(g(z));
```

$$0.7500000000000000000000 - \frac{0.25}{\sqrt{z-z^2}}$$

```
> # We are looking for the largest zero (in (0,1)) of g(z):
> plot(g(z),z=0..1);
```




```
> ## we have an equivalent equation: (3/4)sqrt(z-z^2)=1/4
## or 3sqrt(z-z^2)=1 or 9z-9z^2-1=0 or 9z^2-9z+1=0
## delta= 81-36=45
## solutions are z1= (9-sqrt(45))/18 and z2=
(9+sqrt(45))/18
## we choose the larger one : z2
z2:=evalf((9+sqrt(45))/18);
```

$$z2 := 0.8726779962499649494$$

```
>
```