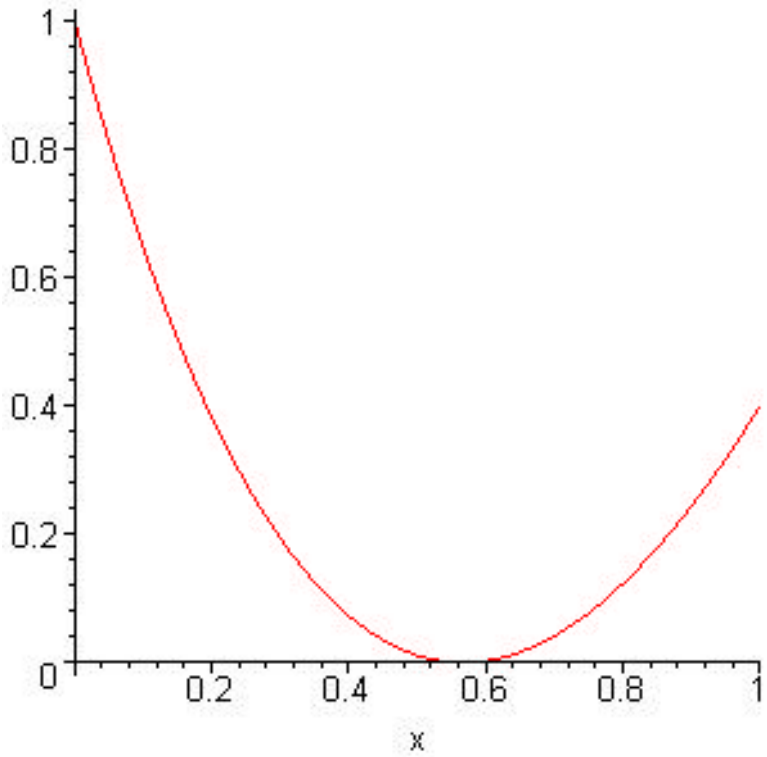
```
> # Assignment #4, Mast 334/ Math 354, Solutions:
>
> # Problems 1a and 2a page 85:
f:=x->x^2-2*x*exp(-x)+exp(-2*x); # x in [0,1]
plot(f(x),x=0..1);
```

$$f := x \to x^2 - 2\,x\,\mathbf{e}^{(-x)} + \mathbf{e}^{(-2\,x)}$$




```
> Digits:=20:
ff:=D(f);
#Newton
print(`Newton Method:`);
MaxError:=0.0000001;
MaxSteps:=30;
a:=0.5;:Er:=100:
for i from 1 to MaxSteps  while (Er >MaxError) and
(abs(a)<10000) do
  anew:=evalf(a-f(a)/ff(a)):
  Er:=abs(a-anew):
  a:=anew:
  print(a, ` error= `,evalf(Er));
end do:
```

$$ff := x \to 2\,x - 2\,\mathbf{e}^{(-x)} + 2\,x\,\mathbf{e}^{(-x)} - 2\,\mathbf{e}^{(-2\,x)}$$

Newton Method:

$$MaxError := 0.1\ 10^{-6}$$

$$MaxSteps := 30$$

$$a := 0.5$$

0.5331555015986090765 4 , *error* = , 0.03315550159860907654

0.5500438056228882775 8 , *error* = , 0.016888304024279201(

0.5585669565504402716 2 , *error* = , 0.008523150927551994(

0.5628484514220416704 5 , *error* = , 0.004281494871601398(

0.5649941998805682474 8 , *error* = , 0.002145748458526577(

0.5660683270089471934 3 , *error* = , 0.0010741271283789459 5

0.5666057041281478772 7 , *error* = , 0.0005373771192006838 4

0.5668744711177672802 8 , *error* = , 0.0002687669896194030 1

0.5670088742252391973 3 , *error* = , 0.0001344031074719170 5

0.5670760806827853365 2 , *error* = , 0.0000672064575461391 9

0.5671096851375916207 1 , *error* = , 0.0000336044548062841 9

0.5671264876715131241 4 , *error* = , 0.0000168025339215034

0.5671348890151046616 7 , *error* = , 0.840134359153753 10^{-5}

0.5671390897060587076 5 , *error* = , 0.420069095404598 10^{-5}

0.5671411900563248386 8 , *error* = , 0.210035026613103 10^{-5}

0.5671422402326543047 7 , *error* = , 0.105017632946609 10^{-5}

0.5671427653211204559 4 , *error* = , 0.525088466151177 10^{-6}

0.5671430278654178232 0 , *error* = , 0.262544297367260 10^{-6}

0.5671431591376029401 3 , *error* = , 0.131272185116931 10^{-6}

0.5671432247736546952 9 , *error* = , 0.656360517551610 10^{-7}

```
> fff:=D(ff);
# Modified Newton
print(`Modified Newton Method:`);
MaxError:=0.000001;
MaxSteps:=30;
a:=0.5;:Er:=100:
for i from 1 to MaxSteps  while (Er >MaxError) and
(abs(a)<10000) do
  anew:=evalf(a-(f(a)*ff(a))/((ff(a))^2-f(a)*fff(a))):
  Er:=abs(a-anew):
```

```
  a:=anew:
  print(a, ` error= `,evalf(Er));
end do:
```

$$fff := x \to 2 + 4\,\mathbf{e}^{(-x)} - 2\,x\,\mathbf{e}^{(-x)} + 4\,\mathbf{e}^{(-2\,x)}$$

Modified Newton Method.

$$MaxError := 0.1\ 10^{-5}$$

$$MaxSteps := 30$$

$$a := 0.5$$

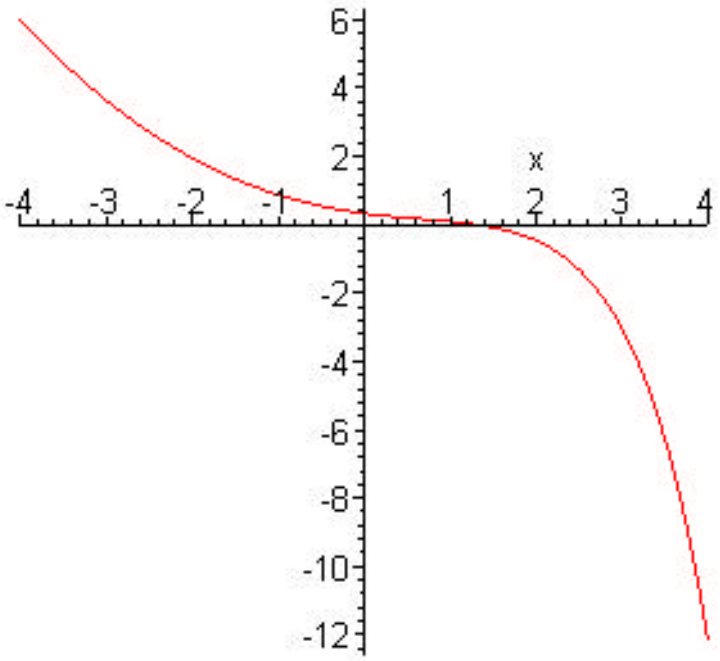
$$0.56801373385948172439\ ,\ error = ,\ 0.06801373385948172439$$

$$0.56714342739319008017\ ,\ error = ,\ 0.00087030646629164422$$

$$0.56714329040969239526\ ,\ error = ,\ 0.13698349768491\ \ 10^{-6}$$

```
> # Problem 10 a page 90:
> # Solve x=(2-exp(x)+x^2)/3 using Steffensen's method and
g:=x->(2-exp(x)+x^2)/3;plot(g(x),x=-4..4);
```

$$g := x \to \frac{2}{3} - \frac{1}{3}\,\mathbf{e}^{x} + \frac{1}{3}\,x^{2}$$




```
> a:=1;# Steffensen's method

print(`Steffensen's method:`);
MaxSteps:=16:Er:=100:MaxError:=.000001:
```

```
for i from 1 to MaxSteps  while (Er >MaxError) and
(abs(a)<10000) do
  a1:=g(a):
  a2:=g(a1):
  anew:=evalf(a-(a1-a)^2/(a2-2*a1+a)):
  Er:=abs(a-anew):
  a:=anew:
  print(a);
end do:
```

$$a := 1$$

Steffensen's method:

0.26409514273021127109

0.25753132511227020207

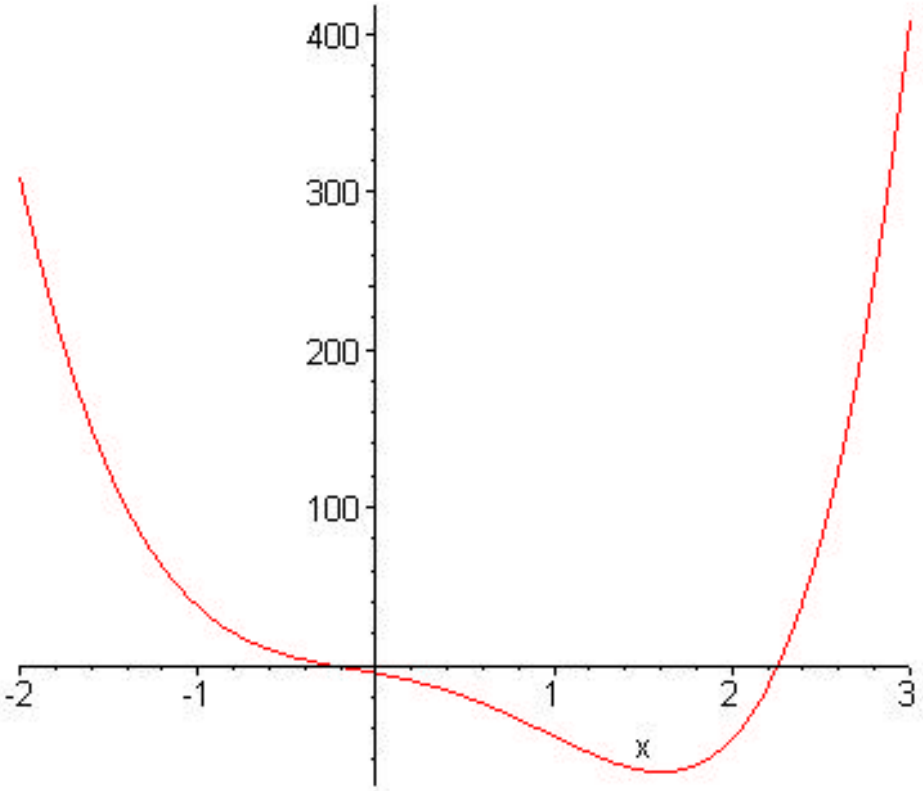
0.25753028543988698007

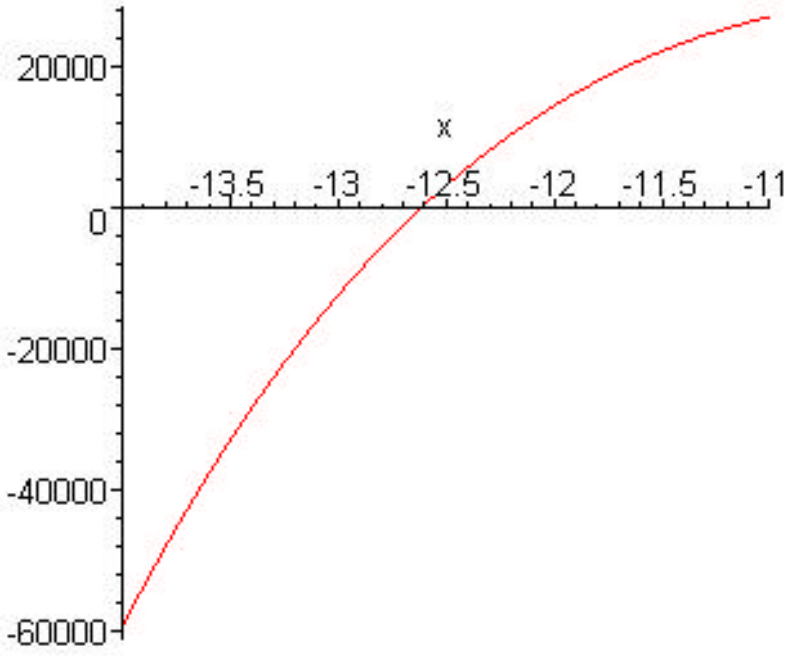
0.2575302854398607604

```
> # problem 2 d page 99:
f:=x->x^5+11*x^4-21*x^3-10*x^2-21*x-5;
ff:=D(f);plot(f(x),x=-2..3);plot(f(x),x=-14..-11);
```

$$f := x \to x^5 + 11\,x^4 - 21\,x^3 - 10\,x^2 - 21\,x - 5$$

$$ff := x \to 5\,x^4 + 44\,x^3 - 63\,x^2 - 20\,x - 21$$


400
300
200
100
-2
-1
1
2
3
x


20000
0
-20000
-40000
-60000
x
-13.5
-13
-12.5
-12
-11.5
-11

> **#Newton**

```
print(`Newton Method:`);
MaxError:=0.0000001;
MaxSteps:=30;
a:=-12.5;:Er:=100:
for i from 1 to MaxSteps  while (Er >MaxError) and
(abs(a)<10000) do
  anew:=evalf(a-f(a)/ff(a)):
  Er:=abs(a-anew):
  a:=anew:
  print(a);
end do:
```

Newton Method:

$MaxError := 0.1\ 10^{-6}$

$MaxSteps := 30$

$a := -12.5$

-12.61650667351734313

-12.61243464599934826

-12.612429524939616726

-12.612429524931524826

```
> a:=0;:Er:=100:
for i from 1 to MaxSteps  while (Er >MaxError) and
(abs(a)<10000) do
  anew:=evalf(a-f(a)/ff(a)):
  Er:=abs(a-anew):
  a:=anew:
  print(a);
end do:
```

$a := 0$

-0.23809523809523809524

-0.25030189828443325145

-0.25023694232014197773

-0.25023694032512699790

```
> a:=2.1;:Er:=100:
for i from 1 to MaxSteps  while (Er >MaxError) and
(abs(a)<10000) do
  anew:=evalf(a-f(a)/ff(a)):
  Er:=abs(a-anew):
```

```
  a:=anew:
  print(a);
end do:
```

$$a := 2.1$$

2.3008053351393731943

2.2619563107858119753

2.2600897115771634570

2.2600855280866080728

2.2600855280656274457

```
> # Now we have to divide f(x) by
(x+12.612429)*(x+0.250236)*(x-2.260085)
P:=x->(x+12.612429)*(x+0.250236)*(x-2.260085);
```

$$P := x \to (x + 12.612429)\,(x + 0.250236)\,(x - 2.260085)$$

```
> sort(expand(P(x)));
```

$$x^3 + 10.602580\,x^2 - 25.914632443281\,x - 7.1330176172530157 40$$

```
> # We redefine f and P so Maple understands that these are
polynomials (not just
# functions):
f:=x^5+11*x^4-21*x^3-10*x^2-21*x-5;
P:=x^3+10.602580*x^2-25.914632*x-7.133017;
```

$$f := x^5 + 11\,x^4 - 21\,x^3 - 10\,x^2 - 21\,x - 5$$

$$P := x^3 + 10.602580\,x^2 - 25.914632\,x - 7.133017$$

```
> result:=quo(f,P,x);
remainder:=rem(f,P,x);
```

$$result := x^2 + 0.397420\,x + 0.700954656400$$

$$remainder := -0.000078519669641200 - 0.000214414567555200\,x + 0.000082228586488000\,x^2$$

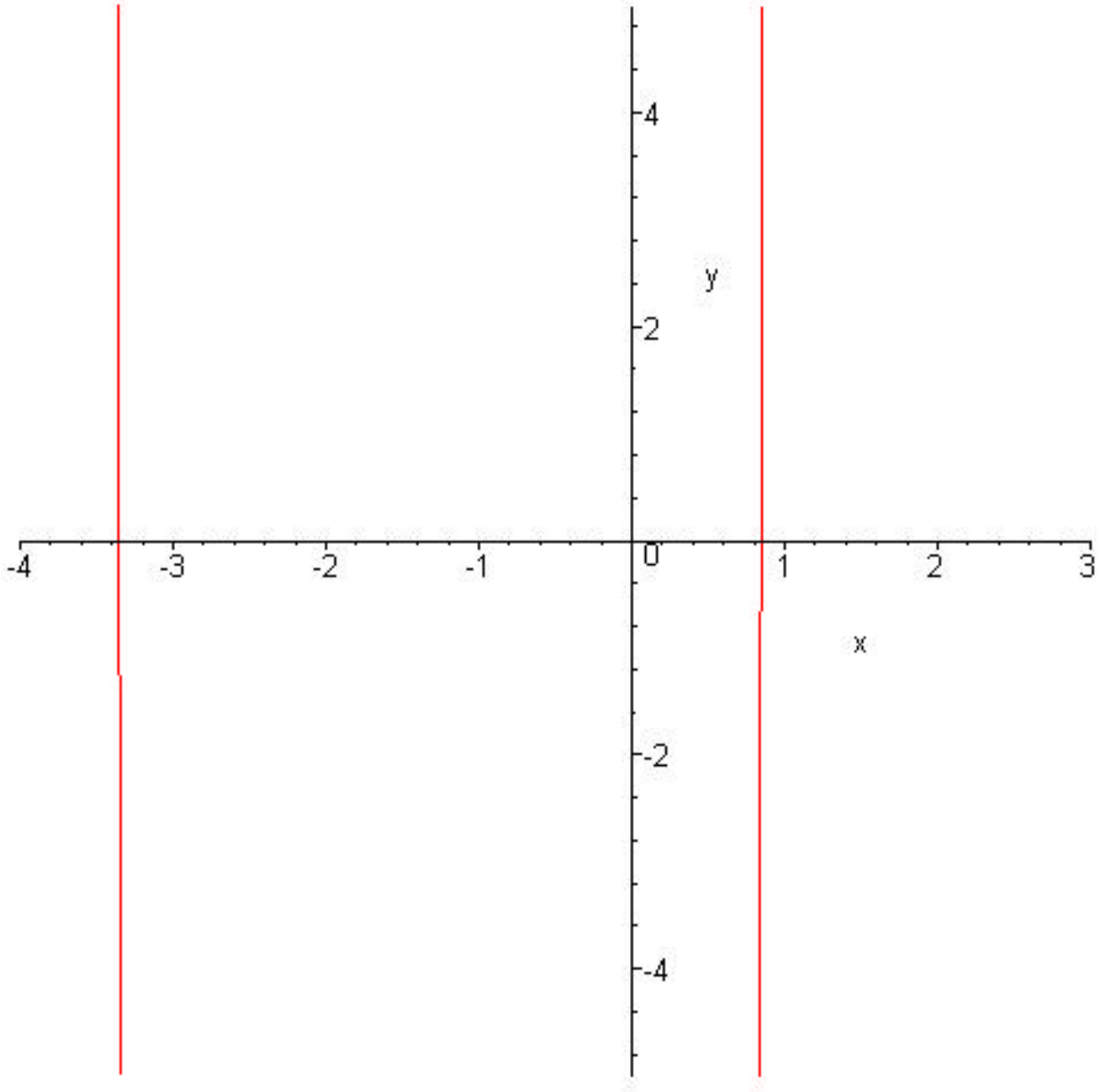
```
> # the division is not perfect but the remainder is small
solve(result=0,x);
```

$$-0.19871000000000000000 + 0.81330744020942043759\,I,\ -0.19871000000000000000 - 0.81330744020942043759\,I$$

```
> # We obtained approximate complex roots of f.


#Problem 2e page 99
f:=x->16*x^4+88*x^3+159*x^2+76*x-240;
plot(f(x),x=-4..3,y=-5..5);
```

$$f := x \to 16\,x^4 + 88\,x^3 + 159\,x^2 + 76\,x - 240$$




```
> ff:=D(f);
#Newton
print(`Newton Method:`);
MaxError:=0.0000001;
MaxSteps:=30;
a:=-3;:Er:=100:
for i from 1 to MaxSteps  while (Er >MaxError) and
(abs(a)<10000) do
  anew:=evalf(a-f(a)/ff(a)):
  Er:=abs(a-anew):
```

```
  a:=anew:
  print(a);
end do:
```

$$ff := x \to 64\,x^3 + 264\,x^2 + 318\,x + 76$$

Newton Method:

$$MaxError := 0.1\ 10^{-6}$$

$$MaxSteps := 30$$

$$a := -3$$

-3.5086956521739130435

-3.3741944773273548810

-3.3582526722068089129

-3.3580445165037971(

-3.3580444814069772[illegible]

```
> #Newton
print(`Newton Method:`);
MaxError:=0.0000001;
MaxSteps:=30;
a:=1;:Er:=100:
for i from 1 to MaxSteps  while (Er >MaxError) and
(abs(a)<10000) do
  anew:=evalf(a-f(a)/ff(a)):
  Er:=abs(a-anew):
  a:=anew:
  print(a);
end do:
```

Newton Method:

$$MaxError := 0.1\ 10^{-6}$$

$$MaxSteps := 30$$

$$a := 1$$

0.8628808864265927977

0.8469445098673862377

0.8467426038165993684[illegible]

0.8467425717222014358[illegible]

```
> # Now we have to divide f(x) by  (x+3.358044)*(x-.846742)
```

```
P:=x->(x+3.358044)*(x-.846742);
```

$$P := x \to (x + 3.358044)\,(x - 0.846742)$$

```
> sort(expand(P(x)));
```

$$x^2 + 2.511302\ x - 2.843396892648$$

```
> # We redefine f and P so Maple understands that these are
polynomials (not just
# functions):
f:=16*x^4+88*x^3+159*x^2+76*x-240;
P:=x^2+2.511302*x-2.843396;
```

$$f := 16\,x^4 + 88\,x^3 + 159\,x^2 + 76\,x - 240$$

$$P := x^2 + 2.511302\ x - 2.843396$$

```
> result:=quo(f,P,x);
remainder:=rem(f,P,x);
```

$$result :=$$

$$16.000000000000000000\ x^2 + 47.819168000000000000\ x + 84.405963763264000000$$

$$remainder := -0.00042025939019545600 - 0.00003459608440972800\ x$$

```
> # the division is not perfect but the remainder is small
solve(result=0,x);
```

$$-1.4943490000000000000 + 1.7442172460456294892\ I,$$

$$-1.4943490000000000000 - 1.7442172460456294892\ I$$

```
> # We obtained approximate complex roots of f.

>
```