```
> ## Assignment 12 , Mast 334/ Math 354 , Solutions

## Problem 7 page 204
> f:=x->exp(2*x)*sin(3*x);
## Derivatives:
d1f:=D(f);d2f:=D(d1f);d3f:=D(d2f);d4f:=D(d3f);
```

$$f := x \to \mathbf{e}^{(2\,x)} \sin(3\,x)$$

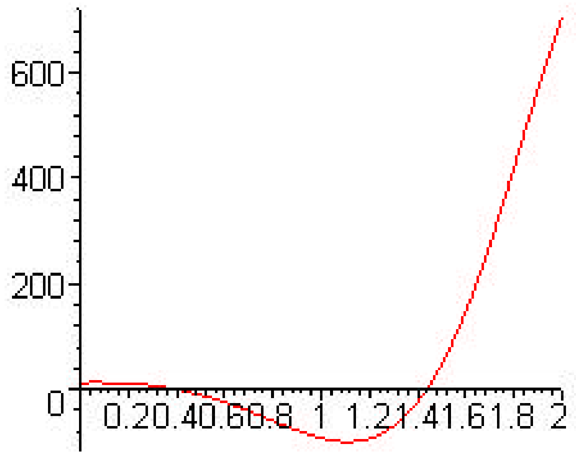
$$d1f := x \to 2\,\mathbf{e}^{(2\,x)} \sin(3\,x) + 3\,\mathbf{e}^{(2\,x)} \cos(3\,x)$$

$$d2f := x \to -5\,\mathbf{e}^{(2\,x)} \sin(3\,x) + 12\,\mathbf{e}^{(2\,x)} \cos(3\,x)$$

$$d3f := x \to -46\,\mathbf{e}^{(2\,x)} \sin(3\,x) + 9\,\mathbf{e}^{(2\,x)} \cos(3\,x)$$

$$d4f := x \to -119\,\mathbf{e}^{(2\,x)} \sin(3\,x) - 120\,\mathbf{e}^{(2\,x)} \cos(3\,x)$$

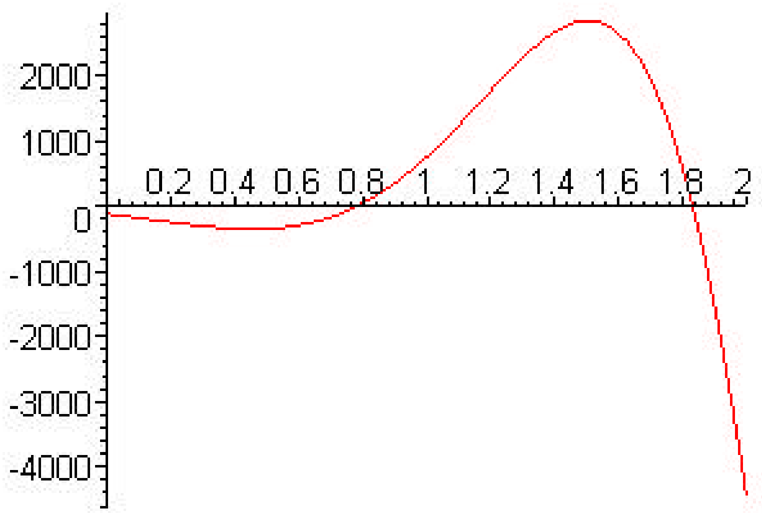
```
> ## Estimate on d2f on [0,2]:
plot(d2f,0..2);
```




```
> ## Thus |d2f|<= d2f(2)
d2f_est:=evalf(d2f(2));
```

$$d2f\_est := 705.3601028$$

```
> ## Estimate on d4f on [0,2]:
plot(d4f,0..2);
```




```
> ## Thus |d4f|<= -d4f(2)
d4f_est:=-evalf(d4f(2));
```

$d4f\_est := 4475.409818$

```
> ## Composite Trapezoidal Rule:
## We want  (b-a)/12 * h^2 *sup( |d2f|) <= 10^(-4)
## or h^2<=10^(-4)*12/(2 * d2f_est)
## or h <=
h_est:=sqrt(10^(-4)*12/(2 * d2f_est));
```

$h\_est := 0.000922295691$

```
> ## To find n we do:
2/h_est;
```

2168.501944

```
> ## Answer: n should be n=2169 (does not have to be even)
> ## and h=1/2169
> ###############################################
## Composite Simpson's Rule:
## We want  (b-a)/180 * h^4 *sup( |d4f|) <= 10^(-4)
## or h^4<=10^(-4)*180/(2 * d4f_est)
## or h <=
h_est:=evalf((10^(-4)*180/(2 * d4f_est))^(1/4));
```

$h\_est := 0.03765758152$

```
> ## To find n we do:
2/h_est;
```

53.11015522

```
> ## Answer: n should be n=54 (it is and has to  be even)
> ## and h=1/54
```

```
> ##################################################
## Composite Midpoint Rule:
## We want  (b-a)/6 * h^2 *sup( |d2f|) <= 10^(-4)
## or h^2<=10^(-4)*6/(2 * d2f_est)
## or h <=
h_est:=evalf((10^(-4)*6/(2 * d2f_est))^(1/2));
```

$$h\_est := 0.000652161537$$

```
> ## To find n we do:
2/h_est;
```

$$3066.724860$$

```
> ## Answer: n should be n=3068 (it  has to  be even)
> ## and h=1/3068
>
> ## Problems 1f,2f,3f,4f page 226
> f:=x->2*x/(x^2-4);
# antiderivative of f(x)  for x  in [1, 1.6] is ln(4-x^2)
exact_value:=evalf(ln(4-1.6^2)-ln(3));
```

$$f := x \to \frac{2\,x}{x^2-4}$$

$$exact\_value := -0.7339691754$$

```
> ## Gaussian quadrature: first we substitute to integrate
over [-1,1]
> h:=x->0.3*x+1.3; # transforms [-1,1] onto [1,1.6]
```

$$h := x \to 0.3\,x + 1.3$$

```
> #Thus int(f(x),1..1.6)=int(f(h(x))*h'(x),-1..1);
## We want to find integral of
g:=x->f(h(x))*0.3;g(x)  ; ## over the interval [-1,1]
```

$$g := x \to 0.3\,\mathrm{f}(\mathrm{h}(x))$$

$$\frac{0.6\,(0.3\,x+1.3)}{(0.3\,x+1.3)^2-4}$$

```
> ## Roots of Legendre polynomials and coefficients are
## given in Table 4.11 page 225
## Gaussian quadrature of order n=2:
x0:=0.5773502692;x1:=-x0;
c0:=1.0;c1:=1.0;
Gaussian_2:=c0*g(x0)+c1*g(x1);
error_2:=abs(Gaussian_2-exact_value);
>
```

$$x0 := 0.5773502692$$

$$x1 := -0.5773502692$$

$$c0 := 1.0$$

$$c1 := 1.0$$

$$Gaussian\_2 := -0.7307230365$$

$$error\_2 := 0.0032461389$$

```
> ## Gaussian quadrature of order n=3:
x0:=0.7745966692;x1:=0.0;x2:=-x0;
c0:=0.5555555556;c1:=0.8888888889;c2:=c0;
Gaussian_3:=c0*g(x0)+c1*g(x1)+c2*g(x2);
error_3:=abs(Gaussian_3-exact_value);
```

$$x0 := 0.7745966692$$

$$x1 := 0.$$

$$x2 := -0.7745966692$$

$$c0 := 0.5555555556$$

$$c1 := 0.8888888889$$

$$c2 := 0.5555555556$$

$$Gaussian\_3 := -0.7337990227$$

$$error\_3 := 0.0001701527$$

```
> ## Gaussian quadrature of order n=4:
Digits:=20;
x0:=0.8611363116;x1:=0.3399810436;
x2:=-x1;x3:=-x0;
c0:=0.3478548451;c1:=0.6521451549;
c2:=c1;
c3:=c0;
Gaussian_4:=c0*g(x0)+c1*g(x1)+c2*g(x2)+c3*g(x3);
error_4:=abs(Gaussian_4-exact_value);
```

$$Digits := 20$$

$$x0 := 0.8611363116$$

$$x1 := 0.3399810436$$

$$x2 := -0.3399810436$$

$$x3 := -0.8611363116$$

$$c0 := 0.3478548451$$

$$c1 := 0.6521451549$$

$$c2 := 0.6521451549$$

$$c3 := 0.3478548451$$

$$Gaussian\_4 := -0.73396039343940008053$$

$$error\_4 := 0.878196059991947\ 10^{-5}$$

```
> ## Gaussian quadrature of order n=5:
x0:=0.9061798459;x1:=0.5384693101;
x2:=0.0;x3:=-x1;x4:=-x0;
c0:=0.2369268805;c1:=0.4786286705;
c2:=0.5688888889;
c3:=c1;c4:=c0;
Gaussian_5:=c0*g(x0)+c1*g(x1)+c2*g(x2)+c3*g(x3)+c4*g(x4);
error_5:=abs(Gaussian_5-exact_value);
```

$$x0 := 0.9061798459$$

$$x1 := 0.5384693101$$

$$x2 := 0.$$

$$x3 := -0.5384693101$$

$$x4 := -0.9061798459$$

$$c0 := 0.2369268805$$

$$c1 := 0.4786286705$$

$$c2 := 0.5688888889$$

$$c3 := 0.4786286705$$

$$c4 := 0.2369268805$$

$$Gaussian\_5 := -0.7339687210956734159$$

$$error\_5 := 0.454304326584021\ 10^{-6}$$

>