

> ## Assignment 11 , Mast 334/ Math 354 , Solutions

Problems 1g,2g,3g,4g,5g,6g page 195

f:=x->x*sin(x);

exact_value:=evalf(int(f,0..Pi/4));

Derivatives:

d1f:=D(f);d2f:=D(d1f);d3f:=D(d2f);d4f:=D(d3f);

$f := x \rightarrow x \sin(x)$

$exact_value := 0.1517464138$

$d1f := x \rightarrow \sin(x) + x \cos(x)$

$d2f := x \rightarrow 2 \cos(x) - x \sin(x)$

$d3f := x \rightarrow -3 \sin(x) - x \cos(x)$

$d4f := x \rightarrow -4 \cos(x) + x \sin(x)$

> # Trapezoidal Rule:

x0:=0;x1:=Pi/4;h:=x1-x0;

Trapezoidal_value:=evalf((h/2)*(f(x0)+f(x1)));

$x0 := 0$

$x1 := \frac{\pi}{4}$

$h := \frac{\pi}{4}$

$Trapezoidal_value := 0.2180895062$

> # Error estimate = $-(h^3/12) * d2f(\theta)$

Since $0 < d2f \leq 2$ we have

error_est:= evalf(-(h^3/12)* 2.0);

actual_error:=exact_value-Trapezoidal_value;

$error_est := -0.0807455122$

$actual_error := -0.066343092$

> # Simpson's Rule:

x0:=0;x1:=Pi/8;x2:=Pi/4;h:=x1-x0;

Simpson_value:=evalf((h/3)*(f(x0)+4*f(x1)+f(x2)));

Error estimate = $-(h^5/90) * d4f(\theta)$

Since $-4 \leq d4f \leq 0$ we have

error_est:= evalf((h^5/90)* 4.0);

actual_error:=exact_value-Simpson_value;

$x0 := 0$

$x1 := \frac{\pi}{8}$

$$x_2 := \frac{\pi}{4}$$

$$h := \frac{\pi}{8}$$

Simpson_value := 0.1513826289

error_est := 0.0004150657618

actual_error := 0.0003637849

> # Midpoint Rule:

x_1:=0;x0:=Pi/8;x1:=Pi/4;h:=x1-x0;

Midpoint_value:=evalf(2*h*f(x0));

Error estimate = (h^3/3)* d2f(theta)

Since 0<=d2f<=2 we have

error_est:= evalf((h^3/3)* 2.0);

actual_error:=exact_value-Midpoint_value;

x_1 := 0

$$x_0 := \frac{\pi}{8}$$

$$x_1 := \frac{\pi}{4}$$

$$h := \frac{\pi}{8}$$

Midpoint_value := 0.1180291903

error_est := 0.04037275610

actual_error := 0.0337172235