```
> ##Assignment #10, Mast 334/ Math 354 , Solutions
#  Problems 1c 3c page 516
# We have T3(x)=4x^3-3x and T4(x)=8x^4-8x^2+1
T3:=x->4*x^3-3*x;
T4:=x->8*x^4-8*x^2+1;
# finding the zeros
(solve(T3(x),x));
```

$$T3 := x \rightarrow 4\,x^3 - 3\,x$$

$$T4 := x \rightarrow 8\,x^4 - 8\,x^2 + 1$$

$$0, \frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}$$

```
> x0:=0;x1:=evalf(sqrt(3)/2);x2:=-x1;# the order does not
matter
```

$$x0 := 0$$

$$x1 := 0.866025404($$

$$x2 := -0.866025404($$

```
> # Building blocks:
L0:=x->(x-x1)*(x-x2)/((x0-x1)*(x0-x2));
L1:=x->(x-x0)*(x-x2)/((x1-x0)*(x1-x2));
L2:=x->(x-x1)*(x-x0)/((x2-x1)*(x2-x0));
```

$$L0 := x \rightarrow \frac{(x - x1)\,(x - x2)}{(x0 - x1)\,(x0 - x2)}$$

$$L1 := x \rightarrow \frac{(x - x0)\,(x - x2)}{(x1 - x0)\,(x1 - x2)}$$

$$L2 := x \rightarrow \frac{(x - x1)\,(x - x0)}{(x2 - x1)\,(x2 - x0)}$$

```
> f:=x->ln(x+2);
f0:=f(x0);f1:=f(x1);f2:=f(x2);
# interpolation:
P2:=x->f0*L0(x)+f1*L1(x)+f2*L2(x);
simplify(expand(P2(x)));
```

$$f := x \rightarrow \ln(x + 2)$$

$$f0 := \ln(2)$$

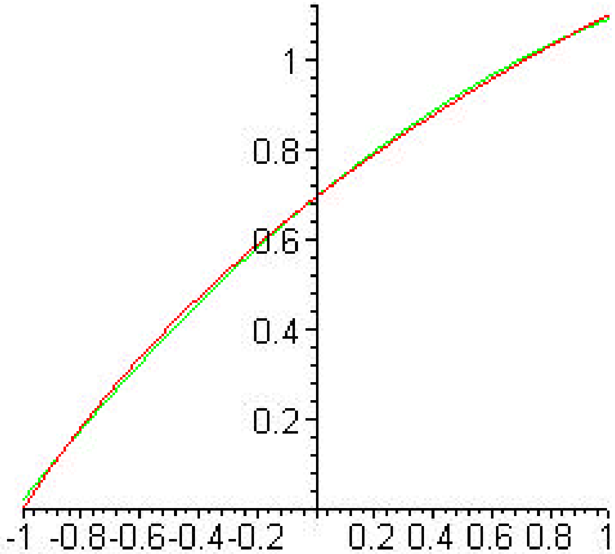
$$f1 := 1.052926193$$

$$f2 := 0.1257288029$$

$$P2 := x \rightarrow f0\,\mathrm{L0}(x) + f1\,\mathrm{L1}(x) + f2\,\mathrm{L2}(x)$$

$$-0.1384262438\;x^2 + 0.6931471806 + 0.5353176625\;x$$

```
> plot([f,P2],-1..1,color=[red,green]);
```



```
> restart:
f:=x->ln(x+2);
## Now interpolation of order 3:
T4:=x->8*x^4-8*x^2+1;
solve(T4(x),x);
```

$$f := x \rightarrow \ln(x+2)$$

$$T4 := x \rightarrow 8\,x^4 - 8\,x^2 + 1$$

$$-\frac{\sqrt{2+\sqrt{2}}}{2},\ \frac{\sqrt{2+\sqrt{2}}}{2},\ -\frac{\sqrt{2-\sqrt{2}}}{2},\ \frac{\sqrt{2-\sqrt{2}}}{2}$$

```
> x0:=evalf(1/2*(2+2^(1/2))^(1/2));
x1:=evalf(1/2*(2-2^(1/2))^(1/2));
x2:=-x0;
x3:=-x1;
```

$$x0 := 0.9238795325$$

$$x1 := 0.3826834325$$

$$x2 := -0.9238795325$$

$$x3 := -0.3826834325$$

```
> # Building blocks:
L0:=x->(x-x1)*(x-x2)*(x-x3)/((x0-x1)*(x0-x2)*(x0-x3));
L1:=x->(x-x0)*(x-x2)*(x-x3)/((x1-x0)*(x1-x2)*(x1-x3));
L2:=x->(x-x1)*(x-x0)*(x-x3)/((x2-x1)*(x2-x0)*(x2-x3));
L3:=x->(x-x1)*(x-x0)*(x-x2)/((x3-x1)*(x3-x0)*(x3-x2));
```

$$L0 := x \rightarrow \frac{(x-x1)\,(x-x2)\,(x-x3)}{(x0-x1)\,(x0-x2)\,(x0-x3)}$$

$$L1 := x \to \frac{(x - x0)\,(x - x2)\,(x - x3)}{(x1 - x0)\,(x1 - x2)\,(x1 - x3)}$$

$$L2 := x \to \frac{(x - x1)\,(x - x0)\,(x - x3)}{(x2 - x1)\,(x2 - x0)\,(x2 - x3)}$$

$$L3 := x \to \frac{(x - x1)\,(x - x0)\,(x - x2)}{(x3 - x1)\,(x3 - x0)\,(x3 - x2)}$$

```
> # interpolation:
f0:=f(x0);f1:=f(x1);f2:=f(x2);f3:=f(x3);
P3:=x->f0*L0(x)+f1*L1(x)+f2*L2(x)+f3*L3(x);
simplify(expand(P3(x)));
plot([f,P3],-1..1,color=[red,green]);
```

$$f0 := 1.072911341$$

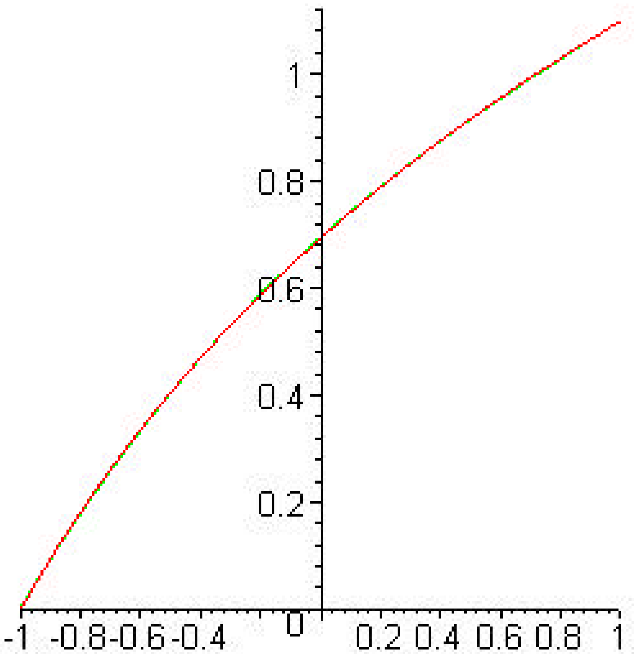
$$f1 := 0.868227345$$

$$f2 := 0.0733624145$$

$$f3 := 0.4807683363$$

$$P3 := x \to f0\ \mathrm{L0}(x) + f1\ \mathrm{L1}(x) + f2\ \mathrm{L2}(x) + f3\ \mathrm{L3}(x)$$

$$0.04909073300\ x^3 - 0.1433460477\ x^2 + 0.6954903832 + 0.4990504197\ x$$




```
> # problem 6 page 517
f:=x->x*exp(x);
P6:=x->x*(1+x+x^2/2+x^3/6+x^4/24+x^5/120);# 6-th Maclaurin
poly
# To obtain T6 we use the recursive formula
```

```
# T{n+1}=2xT{n}-T{n-1}

T3:=x->4*x^3-3*x;
T4:=x->8*x^4-8*x^2+1;
T5:=x->2*x*T4(x)-T3(x);
T6:=x->2*x*T5(x)-T4(x);
```

$$f := x \to x\,\mathbf{e}^x$$

$$P6 := x \to x\left(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5\right)$$

$$T3 := x \to 4\,x^3 - 3\,x$$

$$T4 := x \to 8\,x^4 - 8\,x^2 + 1$$

$$T5 := x \to 2\,x\,\mathrm{T4}(x) - \mathrm{T3}(x)$$

$$T6 := x \to 2\,x\,\mathrm{T5}(x) - \mathrm{T4}(x)$$

```
> # 6th monic Chebyshev polynomial is
TM6:=x->T6(x)/2^5;expand(TM6(x));
```

$$TM6 := x \to \frac{1}{32}\,\mathrm{T6}(x)$$

$$x^6 - \frac{3}{2}x^4 + \frac{9}{16}x^2 - \frac{1}{32}$$

```
> # Economization
P5:=x->P6(x)-TM6(x)*(1/120);expand(P5(x));
```

$$P5 := x \to \mathrm{P6}(x) - \frac{1}{120}\,\mathrm{TM6}(x)$$

$$x + \frac{637}{640}x^2 + \frac{1}{2}x^3 + \frac{43}{240}x^4 + \frac{1}{24}x^5 + \frac{1}{3840}$$

```
> # error made by replacement is at most
evalf((1/120)*(1/2^5));
```

0.000260416666'

```
> # original error |f-P6| is less than |f^(7)(theta)/7! *
x^7|<=|f^(7)(theta)|/7!
d7f:=(D@@7)(f);# 7-th derivative of f
```

$$d7f := x \to 7\,\mathbf{e}^x + x\,\mathbf{e}^x$$

```
> ## we see that  |f^(7)(theta)|/7!<= 8exp(1)/7!
evalf(8*exp(1)/7!);
```

0.004314733059

```
> # total error is
.4314733059e-2+.2604166667e-3;
```

0.00457514972(

```
> #Perhaps we can economize more subtracting (1/24)TM5
# The extra error would be at most
evalf((1/24)*(1/2^4));
```

$$0.002604166667$$

```
> # YES !!! the sum still less than 0.01
TM5:=x->T5(x)/2^4;expand(TM5(x));
```

$$TM5 := x \to \frac{1}{16}\,\mathrm{T5}(x)$$

$$x^5 - \frac{5}{4}x^3 + \frac{5}{16}x$$

```
> P4:=x->P5(x)-(1/24)*TM5(x);expand(P4(x));
```

$$P4 := x \to \mathrm{P5}(x) - \frac{1}{24}\,\mathrm{TM5}(x)$$

$$\frac{379}{384}x + \frac{637}{640}x^2 + \frac{53}{96}x^3 + \frac{43}{240}x^4 + \frac{1}{3840}$$

```
> # The total error is now:
.4575149726e-2+.2604166667e-2;
```

$$0.007179316393$$

```
> # If we want to reduce the order further, the extra error
would be:
> evalf((43/240)*(1/2^3));
```

$$0.02239583333$$

```
> # So it is too big. Thus, P4 is the polynomial we were
looking for.
```