Solutions to Midterm Examination
MATH 205: Calculus II

Instructor: Dr. Ming Mei

Solution to Q1: a). Let $u = e^x$, then $du = e^x dx$, and

$$\int e^x \sin(e^x) \mathrm{dx} = \int \sin u \mathrm{du} = -\cos u + C = -\cos(e^x) + C.$$

b). Set $u = 4 + 3x$, then $du = 3dx$ and

$$\int_0^7 \sqrt{4+3x} \mathrm{dx} = \frac{1}{3} \int_4^{25} \sqrt{u} \mathrm{du} = \frac{2}{9} u^{3/2} \Big|^2 5_{u=4} = 26.$$

c).

$$\int (\ln x)^2 \mathrm{dx} = x(\ln x)^2 - 2 \int \ln x dx$$

[integration by parts: $u = (\ln x)^2,\ dv = dx, \Rightarrow du = (2\ln x)/x,\ v = x$]

$$= x(\ln x)^2 - 2\Big(x \ln x - \int dx\Big)$$

[integration by parts: $u = \ln x,\ dv = dx, \Rightarrow du = 1/x,\ v = x$]

$$= x(\ln x)^2 - 2x \ln x + 2x + C.$$

d).

$$\int_{\sqrt{\pi/2}}^{\sqrt{\pi}} \theta^3 \cos(\theta^2) \mathrm{d}\theta = \frac{1}{2} \int_{\pi/2}^{\pi} x \cos x dx$$

[substitution: $x = \theta^2 \Rightarrow dx = 2\theta d\theta$]

$$= \frac{1}{2} x \sin x \Big|_{x=\pi/2}^{\pi} - \frac{1}{2} \int_{\pi/2}^{\pi} \sin x dx$$

[integration by parts: $u = x,\ dv = \cos x dx \Rightarrow du = dx,\ v = \sin x$]

$$= \frac{1}{2} x \sin x \Big|_{x=\pi/2}^{\pi} + \frac{1}{2} \cos x \Big|_{x=\pi/2}^{\pi} = -\frac{1}{2} - \frac{\pi}{4}.$$

Solution to Q2: Since

$$\frac{1}{b-0}\int_0^b (2+6x-3x^2)dx = 3,$$

namely,

$$\frac{1}{b}(2x+3x^2-x^3)\Big|_0^b = 3,$$

we then have

$$2+3b-b^2 = 3,$$

which solves $b_{1,2} = \frac{3\pm\sqrt{5}}{2}$.

Solution to Q3: The parabola $y = 1 - x^2$ intersects the x-axis at the points $(-1, 0)$ and $(1, 0)$.
a). The area of R is

$$A = \int_{-1}^1 (1-x^2)dx = 2\int_0^1 (1-x^2)dx = 2\Big(x - \frac{x^3}{3}\Big)\Big|_0^1 = \frac{4}{3}.$$

b). The volume of the solid obtained by rotating R about the x-axis is

$$V = \int_{-1}^1 \pi(1-x^2)^2 dx = 2\pi\int_0^1 (1-2x^2+x^4)dx = 2\Big(x - \frac{2x^3}{3} + \frac{x^5}{5}\Big)\Big|_0^1 = \frac{16}{15}\pi.$$