

**Fall 2004, CONCORDIA UNIVERSITY**  
**MAST 334/MATH 354/MATH 618F: Numerical Analysis**

**Sample Questions to Midterm Test**

Instructor: Dr. Ming Mei

1. [10pts] Given an equation  $x \ln x = \ln 3$  in  $[1, 2]$ .
  - a). Prove that it has a unique solution in  $[1, 2]$ .
  - b). To find the solution numerically by Bisection method with accuracy  $10^{-4}$ , how many steps should be carried out?
  - c). Given two iterative schemes

$$i). x_{n+1} = \frac{\ln 3}{\ln x_n}, \quad ii). x_{n+1} = x_n - x_n \ln x_n + \ln 3,$$

which one will work well? why?

- d). Write an iterative scheme by Newton's method and test its convergent order.
2. Given nodes  $x_0 = 0$ ,  $x_1 = 0.5$ ,  $x_2 = 1$ , and function  $f(x) = e^{2x}$ .
    - a). Find its Lagrange interpolation  $P_2(x)$  on these nodes, and estimate  $|f(x) - P_2(x)|$ .
    - b). Write its Newton's divided-difference polynomial  $\bar{P}_2(x)$ .
    - c). Write its Hermite interpolation  $H_5(x)$  on  $x_0, x_1, x_2$ , and estimate  $|f(x) - H_5(x)|$ .

3. Let

$$S(x) = \begin{cases} S_0(x) = 1 + 2x - x^3, & \text{if } 0 \leq x < 1, \\ S_1(x) = 2 + b(x-1) + c(x-1)^2 + d(x-1)^3, & \text{if } 1 \leq x \leq 2. \end{cases}$$

- a). If  $S(x)$  is a natural cubic spline interpolation, determine  $a$ ,  $b$  and  $c$ .
- b). If  $S(x)$  is a clamped cubic spline interpolation, determine  $a$ ,  $b$  and  $c$ .